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INTERIOR BALLISTICS

by

M. E. Serevryakov

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SECTION IV.- THE PHYSICAL CONCEPTS OF PYRODYNAMICS

CHAPTER 1 - THE PHENOMENON OF A SHOT AND ITS BASIC RELATIONS

Pyrodynamics is the study of the phenomena occurring in the bore of a gun when the latter is fired, and a means for establishing the relation existing between the loading conditions and the various physical and chemical processes and mechanical phenomena occurring thereby.

The mutual relationship and interdependence of the various elements and factors involved are clearly manifested in the phenomenon of a shot. For example, the movement of a shell depends on gas pressure, whereas the pressure itself depends on both the burning of powder and the initial air space back of the shell, the latter in turn depending on the speed of the shell.

The phenomenon of a shot may be considered to consist of the following periods.

1. PRELIMINARY PERIOD

The action of a negligible external impulse - such as the percussion of a firing pin or heating by an electric current - ignites the composition of a percussion cap, and the resulting flame in turn ignites the igniter mixture in the primer cup (usually in the form of a tablet of black powder). The gases produced by the igniter and the incandescent particles of its combustion products enter the powder chamber through a special opening, and the resulting high temperature and pressure (p_B = 20-50 kg/cm^2) cause the ignition of the powder charge.

When ignited, the powder burns at first in a constant volume until the gas pressure becomes sufficiently high to overcome the resistance of the rotating band and force it into the rifling grooves.

This period of a shot may be considered as being purely pyrostatic in character, because the powder burns in a constant volume (space).

Inasmuch as the rifling is provided with a forcing cone at its start, the rotating band enters the grooves gradually, and upon attaining the full depth of the thread its resistance undergoes a sudden drop and the shell proceeds through the bore with the band already fully notched.

The force Π_0 necessary for notching the band to the full depth of the grooves taken with relation to the cross-sectional unit area of the bore s, i.e., Π_0/s, is called the "pressure necessary to overcome the inertia of the projectile" and is designated as $p_0 = \Pi_0/s$ kg/cm^2.

Pressure p_0 may vary from 250 to 500 kg/cm^2 depending on the design of the rotating band and the rifling in the bore.

This period of a shot, when the powder gases commence to move the projectile and overcome the increasing resistance of the band until the latter is notched to the full depth of the grooves and traverses a specified distance, may be called the "forcing period" or the period of notching of the band. During this period the projectile traverses a distance equal to that measured from the initial position of the rear edge of the rotating band to the point at which the rifling grooves attain their full depth.

This period is considerably more complex than the pyrostatic period and is more difficult to analyze. Inasmuch as the initial chamber dimensions undergo a very small change during this displacement of the projectile, both periods are usually combined into a single preliminary period for the sake of simplicity, by assuming that the wedging of the band into the rifling occurs instantaneously and that the movement of the projectile commences as soon as the gas pressure equals the pressure p_0 (i.e., the pressure to overcome the inertia of the projectile).

This period is called the "preliminary period"; the pressure varies from 1 to p_B and then to p_0 and the change occurs during the period t_0.

In fig. 1 this period is depicted by the curve segment ab and the time t_0; in fig. 2, the element corresponding to it is the segment o-p_0 on the ordinate.

Methods are now available for the solution of the problem of interior ballistics which take into consideration the gradual breaking in of the rotating band into the rifling of the bore. These methods will be considered later.

2. FIRST PERIOD

The preliminary period is followed by the basic or first period of a shot, by the period of burning of powder and gas formation in a variable space, during which the powder gases impart a velocity to the projectile and thus perform the work at the expense of the energy confined in them and overcome a series of resistances.

This period, measured from the start of the projectile's movement until the end of powder burning in the bore, when the inflow of fresh gases stops, is the most complicated period: on the one hand the process of burning and the continuous inflow of gases increase the pressure inside the bore, whereas on the other hand the continuous acceleration of the projectile and the resulting increase of the initial "air space" tend to reduce this pressure.

At the start of the basic period, when the velocity of the projectile is still not very high, the quantity of gases increases at a greater rate than the volume of the initial air space, and the pressure increases until it reaches a maximum value p_m. However, the pressure increase and hence the increased acceleration of the projectile cause a rapid increase of the air space (volume) back of the projectile, so that notwithstanding the continued burning of the powder and the inflow of fresh gases, the pressure begins to drop until it attains a value p_K at the end of burning; at the same time the velocity of the projectile increases from zero to v_K. The powder gases perform most of their work during this basic period.

The maximum gas pressure is also developed during this period - this constitutes one of the fundamental ballistic characteristics of the powder and the gun in firing.

The maximum pressure serves as the basic data for establishing the wall thickness of the gun barrel and the projectile, whereas a knowledge of the associated maximum acceleration of the projectile is necessary for designing the inertia parts of time fuzes and firing devices.

3. SECOND PERIOD

The inflow of fresh gases stops at the end of burning of the powder, but inasmuch as the remaining gases still possess a very high reserve of energy, they continue to expand without an inflow of energy while the projectile completes its remaining path in the bore (up to the muzzle face), and thus continue to perform work and increase the velocity and kinetic energy of the projectile. This period constitutes a physical process in which a definite quantity of highly compressed and heated gases undergo expansion. Inasmuch as the velocity of the projectile is already high at the end of burning and continues to increase further, the projectile traverses the remaining portion of its path very rapidly. The ensuing heat losses through the walls of the gun barrel may be therefore disregarded and the entire period may be considered as "the period of adiabatic expansion of the gases." It is called the second period and terminates at the instant the base of the projectile passes the muzzle face of the gun barrel. The pressure drops from p_K to p_0, whereas the velocity of the projectile increases from v_K to $v_Д$ (see figs. 1 and 2).

Both periods occur during a very short period of time - varying from 0.001 to 0.060 second, depending on the length and caliber of the gun barrel.

If we introduce the designations:

s - cross-sectional area of bore;

p - gas pressure inside the bore at a given instant;

ℓ - distance traversed by projectile;

m - mass of projectile;

v - velocity of projectile,

then, according to the general theory of mechanics, to wit, that "the increment of work done by a force equals the increment of kinetic energy," we will have(*):

$$psd\ell = d\left(\frac{mv^2}{2}\right).$$

Integrating, we get:

$$s\int_0^{\ell} pd\ell = \frac{mv^2}{2},$$

whence

$$v = \sqrt{\frac{2s}{m}\int_0^{\ell} pd\ell}.$$

The expression $\int_0^{\ell} pd\ell$ represents the area confined between the abscissa ℓ and the pressure curve; and inasmuch as it continuously increases, the velocity v will also undergo a continuous increase, whereby the nature of the increase of v will depend on the characteristic of the pressure curve. Inasmuch as the pressure, after reaching a maximum, undergoes a continuous drop, the area increase becomes smaller and smaller, and the velocity increment of the projectile gradually decreases at the end of its travel through the bore, i.e., the v,ℓ curve becomes flatter.

(*) ps - the product of pressure by the area equals the force applied to the entire area of the projectile's base.

According to the equation of the projectile's motion

$$ps = m\frac{dv}{dt},$$

the pressure curve drawn to a specific scale gives the curve of the projectile's accelerations:

$$\frac{dv}{dt} = \frac{s}{m} p,$$

where dv/dt is the tangent of the angle of inclination of the curve representing the velocity of the projectile as a function of time.

Inasmuch as the pressure continues to increase until it reaches its maximum, the velocity curve v, t proceeds with an increasing angle of inclination with its convex side directed downward. A point of inflexion is obtained at the point of maximum pressure, and thereafter, as the pressure decreases, the v, t curve continues with its convex side directed upwards:

$$v = \frac{s}{m}\int_{t_0}^{t} pdt.$$

4. THIRD PERIOD

After the projectile leaves the barrel, the gases flowing behind it with a high velocity continue to exert a pressure on the base of the projectile for a certain distance ℓ_n, and thus continue to accelerate the projectile. This period of a shot is called the third period or the "after-effect period of the gases." The projectile acquires its maximum velocity v_{max} at the end of this third period, following which its velocity begins to decrease under the action of air resistance.

In addition to this after-effect action on the projectile, the gases exert pressure also on the gun barrel; the latter action plays an important part in the design of the gun mount, and the fuzes. The duration of the after-effect of the gases on the gun mount is considerably longer than on the projectile.

In addition to the basic processes mentioned above, there is also a series of auxiliary processes affecting the phenomenon of a shot. Thus, for example, the movement of the projectile through the bore is accompanied by a non-uniform displacement of the gases in the initial air space and also by the recoil of the barrel. The projectile acquires a rotary or spinning motion in addition to the forward straight-line motion. Some of the gases escape through the clearance between the rotating band and the rifling of the bore, thus overtaking the projectile without first performing useful work; a portion of the heat energy is spent on heating of the barrel walls (losses due to heat transfer).

The following basic processes and relationships can therefore be established on the basis of the shot phenomenon discussed above.

1) The source of energy is derived from the expanding gases formed during the burning of the powder, and hence the laws of gas formation constitute the basic relationships expressing the process of burning of powder. The following laws apply to the science of pyrostatics:

a) gas formation governed by the burned thickness of the powder

$$\psi = \varkappa z(1 + \lambda z + \mu z^2),$$

or

$$\psi = \varkappa_1 z(1 + \lambda_1 z),$$

where

$$z = \frac{e}{e_1} \quad \text{and} \quad \psi = \frac{\Lambda c\Gamma}{\Lambda_1};$$

b) burning rate

$$u = u_1 p;$$

c) rate of gas formation

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1 p = \frac{\alpha}{I_K}\frac{S}{S_1}p.$$

The following relations are used in the case of the physical law of burning: $\psi = f(I)$ or $I = F(\psi)$, and also $d\psi/dt = \Gamma p$.

2) The gases formed during the burning of powder contain a large supply of heat energy; a portion of this energy is transformed into work when the gun is fired, which work is utilized mainly to impart kinetic energy to the projectile, the charge and the barrel and partly to overcome parasitic resistances. A portion of the heat is absorbed by the walls of the gun barrel. A major part of the

energy is not used up however, and is ejected from the bore in the form of very hot gases after the projectile leaves the barrel.

Inasmuch as a shot is accompanied by transformation of energy, the first law of thermodynamics, i.e., the law of conservation of energy, gives the second basic relationship.

It is written thus:

$$Q = U + A\Sigma L,$$

where Q - quantity of heat supplied to the system from the exterior;

U - internal energy of powder gases;

ΣL - total amount of exterior work done by gases, including the work required to overcome parasitic resistances;

$\frac{1}{A} = E$ - mechanical equivalent, equal to 4270 kg · dm/cal.

This fundamental relationship is transformed in pyrodynamics into the so-called fundamental equation of pyrodynamics (see below).

3) The next fundamental relationship is the equation depicting the translation of the projectile.

It can be written two ways:

a) the first form of the equation of motion (Newton's law)

$$ps = m\frac{dv}{dt};$$

b) the second form of the equation of motion

$$sp = mv\frac{dv}{dt},$$

where s - cross-sectional area of the bore;

p - gas pressure;

$m = q/g$ - mass of projectile;

v - velocity of projectile;

l - path traversed by projectile.

Other theorems and relationships of mechanics will be introduced later in the text in addition to the three fundamental relationships specified above.

4) Inasmuch as the charge-projectile-barrel system is brought into motion when a shot is fired by the action of the internal forces, i.e., by the pressure exerted by the powder gases, the following theorem of mechanics can be applied to it in the case of free recoil: "If a system is subjected to the action of internal forces, the displacement of its separate parts is such that the sum of the quantities of motion (the sum of moments) equals zero:"

$$mv + \mu U + MV = 0,$$

where M and V - mass and velocity of the recoiling parts;

μ and U - mass and velocity of charge.

This gives the relation between the velocity of the projectile and the velocity of the recoiling parts.

5) The equation of rotary motion of the projectile is obtained from the theorem: "The moment of a couple equals the moment of inertia multiplied by the angular acceleration":

$$rN = J\frac{d\Omega}{dt},$$

where r - distance measured from the axis of the projectile to the center of the driving edge;

N - turning force;

J - moment of inertia of projectile relative to the axis of rotation;

Ω - angular velocity;

$\frac{d\Omega}{dt}$ - angular acceleration.

CHAPTER 2 - ENERGY EQUILIBRIUM WHEN A SHOT IS FIRED

When a shot is fired, a considerable portion of the energy developed by the powder gases is spent on performing work and is converted into kinetic energy of motion of the projectile. Furthermore, a part of the energy is expended on the performance of other work of lesser magnitude which must be taken into consideration, to obtain a full analysis of the equilibrium of energy when a shot is fired.

Say, a portion ψ of a charge ω is burned at the instant t, at which time a projectile whose weight is q has traversed a distance ℓ with a velocity v; the temperature of the burning powder is T_1. Inasmuch as the gases had performed work at the given instant and had cooled off, we shall designate their mean temperature by T, where $T < T_1$.

$Q\omega\psi$ cal of heat are evolved during the burning of $\omega\psi$ kg of powder, which quantity of heat is equivalent to work $\ni Q\omega\psi$, where $\ni = 4270$ kg · dm/cal - the mechanical heat equivalent.

If we designate the mean heat capacity at constant volume for temperature T_1 by c_{w1}, $Q = \bar{c}_{w1}T_1$ and

$$E_1 = \ni \bar{c}_{w1} T_1 \omega\psi \text{ kg} \cdot \text{dm}.$$

This energy would be fully transformed into work, if the gas temperature were lowered to absolute zero.

Actually, this quantity of gas, having accomplished the work of moving the projectile and a series of other secondary items of work at the instant t, cools down only to a certain temperature $T < T_1$ and hence continues to retain a supply of unexpended energy equal to

$$E = \ni \bar{c}_w T \omega \psi \text{ kg} \cdot \text{dm},$$

where $\bar{c}_w$ is the mean heat capacity for temperature T.

Hence the energy expended at the instant t on the performance of external work will be expressed by the difference

$$E_1 - E = \ni \bar{c}_{w1} T_1 \omega \psi - \ni \bar{c}_w T \omega \psi.$$

Upon performing elementary transformation, we get:

$$E_1 - E = \ni \omega \psi \left[\left(A + b\frac{T_1}{2} \right) T_1 - \left(A + b\frac{T}{2} \right) T \right] = \ni \omega \psi \Big[A(T_1 - T) + $$
$$+ \frac{b}{2} (T_1^2 - T^2) \Big] = \ni \omega \psi (T_1 - T) \left(A + b\frac{T_1 + T}{2} \right) = \ni \Big|_{T_1}^{T} c_w (T_1 - T) \omega \psi,$$

where $\Big|_{T_1}^{T} c_w = A + b\frac{T_1 + T_2}{2}$ is the true heat capacity corresponding to the mean temperature in the interval between T_1 and T.

When a shot is fired, the gas temperature varies from T_1 to T_2, corresponding to the instant the projectile passes the face of the muzzle. The temperature interval is the one that is of practical value to interior ballistics.

Since the b coefficient is small, the change of $\Big|_{T_1}^{T} c_W$ is small, and its average value can be considered to be constant for the entire process of the projectile's motion along the bore, i.e.,

$$\Big|_{T_1}^{T_A} c_W = A + b\frac{T_1 + T_A}{2};$$

we shall designate it by c_W' for short.

The graph in fig. 87 shows the variation of c_W with temperature.

$$Q_1 = \left(A + b\frac{T_1}{2}\right)T_1;$$

$$Q = \left(A + b\frac{T_1 + T}{2}\right)(T_1 - T);$$

the heat capacity c_W and the quantity of heat Q relate to a unit of gas by weight.

This graph shows that the heat quantity Q corresponding to a specific temperature difference is greater at high temperatures approaching T_1, than at low temperatures, because of the increased heat capacity.

According to the first law of thermodynamics, the energy balance when a shot is fired can be written as follows:

$$\ni c_W'(T_1 - T)\omega\psi = \Sigma E_i,$$

where $\ni$ - mechanical heat equivalent (427 kg-m/kcal)

ΣE_i - total amount of work done by the gases when a shot is fired, including the work necessary to overcome parasitic resistances.

GRAPHIC NOT REPRODUCIBLE

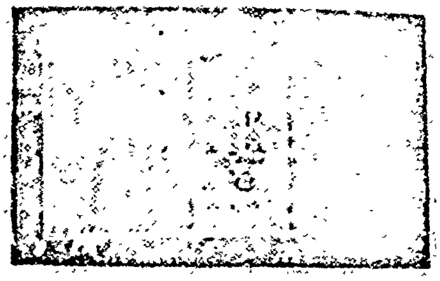

Fig. 87 - The Dependence of Heat Capacity of Gas on Temperature.

When a shot is fired, the energy confined in the gases is expended on the performance of the following forms of work:

1) Energy E_1 for the translation of the projectile, measured by the magnitude of the kinetic energy $mv^2/2$, is the basic form of energy expended during the movement of the projectile through the bore of the gun.

2) Energy E_2 is expended on the rotary motion of the projectile.

3) Energy E_3 is expended to overcome the frictional resistance between the rotating band of the projectile and the walls of the bore (bore + rifling grooves), and also for overcoming the friction between the walls of the projectile and the lands (of the rifling).

4) Energy E_4 is expended on the displacement of the gases of the charge itself and of the unburned portion of powder.

5) Energy E_5 is expended on the displacement of the recoiling parts and is measured by their kinetic energy $MV^2/2$.

6) Energy E_6 is used to force the rotating band of the projectile into the rifling grooves.

7) Energy E_7 is expended for heating the walls of the barrel, shell case and shell when the gun is fired - energy lost on heat transfer.

8) Energy E_8 is confined in the gases escaping through the clearances between the rotating band and the walls of the barrel.

9) Energy E_9 is expended on overcoming the air resistance and on the displacement of the air column present in the bore ahead of the projectile.

Of the above nine forms of expended energy, the first five must be accounted for directly, E_6 is accounted for directly or indirectly; E_7 is a form of heat energy which cannot be easily determined, and is accounted for indirectly for lack of a sufficiently satisfactory theory and test data to permit determining the heat lost to the walls of the barrel. The quantity of gas escaping through the clearances formed between the rotating band and the walls of the bore cannot be computed and has a random value; therefore, energy E_8 corresponding to it is not taken into consideration. This applies also to energy E_9 which is small in comparison with the other energy values.

The secondary work items will be discussed later. We shall note here (without offering proof) that E_2, E_3, E_4, E_5 are proportional to the main form of the work done by the powder gases, i.e., $E_1 = mv^2/2$. Hence, if each of these four forms of work is represented in the form

$$E_i = k_i \frac{mv^2}{2},$$

where k_i = proportionality factors, determined by means of formulas which will be presented later, then the total amount of expended energy accounted for directly can be expressed in the form:

$$\sum_{1}^{5} E_i = \sum_{1}^{5} k_i \frac{mv^2}{2} = \frac{mv^2}{2} (1 + k_2 + k_3 + k_4 + k_5).$$

The sum of the coefficients in this expression is denoted by φ:

$$\varphi = 1 + k_2 + k_3 + k_4 + k_5.$$

The φ coefficient takes into account the secondary work items, and its numerical value for conventional type weapons varies between 1.05 and 1.20 depending on the loading conditions, and may exceed these values.

Thus, assuming that the expended values of energies E_6 and E_7 will be accounted for indirectly, the equation of energy balance during a shot will have the following form:

$$\partial \bar{c}_W T_1 \omega \psi - \partial \bar{c}_W T \omega \psi = \frac{\varphi m v^2}{2}.$$

This equation shows that the difference between two thermal conditions of the powder gases has become converted into a sum of external work items, where all the secondary work items are taken care of by the coefficient $\varphi > 1$. If this coefficient referred only to the mass m, rather than to the entire kinetic energy $mv^2/2$, we could assume that the work is performed by the gases for the purpose of imparting translation with the same velocity v to a heavier projectile of mass φm.

Thus, by introducing the coefficient φ, the actual motion of the projectile with the secondary work done by the gases taken into consideration, is replaced by a condition involving only the translation with the same velocity of a heavier projectile having a fictitious mass φm(*). The energy expended thereby remains the same. Coefficient φ is called the "fictitious mass coefficient."

The introduction of this fictitious magnitude helps to simplify without the introduction of an appreciable error calculations involving complex formulas.

It would be more correct to call φ the "secondary work coefficient," because this value depicts the relationship between the main and secondary work items (where the main work is taken to be equal to unity).

Derivation of the Fundamental Equation of Pyrodynamics

The energy balance equation depicts the relationship between the burned portion of the charge ψ, velocity of the projectile v, and the temperature of the gases formed at the given instant in the initial air space. Neither the length of travel ℓ of the projectile, nor the gas pressure p enters this equation. Nevertheless the basic problem of pyrodynamics is that of finding the relation between the distance ℓ traversed by the projectile, its velocity v and pressure

(*) The concept of a fictitious mass was introduced for the first time by Prof. N.A. Zabudsky, "DAVLENIYE POROKHOVYKH GAZOV V KANALE 3-DM PUSHKI" (Pressure Developed by Powder in the Bore of a 3-inch Cannon), 1894.

p exerted by the gases on the projectile and the walls of the bore. Therefore the energy balance equation must be transformed in such a manner that it would depict the connection between the above-mentioned values p, v and l.

We know from thermodynamics that

$$c_p - c_w = AR = \frac{R}{\ni},$$

whence

$$\ni = \frac{R}{c_p - c_w}$$

$$\ni c_w = \frac{Rc_w}{c_p - c_w} = \frac{R}{\frac{c_p}{c_w} - 1} = \frac{R}{k - 1},$$

where R - gas constant;

$c_p/c_w = k$ - heat capacities ratio (adiabatic index).

We shall introduce for the sake of simplicity the denotation $k - 1 = \theta$; then:

$$\ni c_w = \frac{R}{\theta};$$

$$\theta = \frac{c_p - c_w}{c_w} = \frac{A_2 + bT - A_1 - bT}{A_1 + bT} = \frac{A_2 - A_1}{A_1 + bT}.$$

θ is a gradually decreasing function of temperature. When the gas temperature varies from T_1 to T_x, the mean value θ' will correspond to the mean temperature $T_{cp} = \frac{T_1 + T_n}{2}$ and the corresponding value of c_w':

$$\theta' = \frac{A_2 - A_1}{A_1 + b\frac{T_1 + T}{2}} = \frac{1}{A + BT_{cp}} .$$

Then

$$\partial c_w' = \frac{R}{\theta'} .$$

Substituting this expression in the energy balance equation, we will get:

$$\frac{R}{\theta} T_1 \omega\psi - \frac{R}{\theta} T\omega\psi = \frac{\varphi m v^2}{2} .$$

In order to exclude from this equation the variable T, we shall replace the expression $RT\omega\psi$, using as the basis the equation depicting the condition of the gas powders at the given instant, corresponding to the burned portion of the charge ψ:

$$pW = RT\omega\psi, \tag{51}$$

where W is the free space in the initial air space (back of the projectile) at the given instant:

$$W = W_0 + s\ell - \alpha\omega\psi - \frac{\omega}{\delta}(1 - \psi) = W_\psi + s\ell;$$

here W_ψ - free space in the powder chamber at the instant the portion of the charge ψ is burned in it;

s - cross-sectional area of the barrel;

$s\ell$ - added volume when the projectile had traversed the distance ℓ.

When using equation (51), it should be borne in mind that it applies to specific quantities of gas in a stationary condition, whereas we make the assumption that this equation is valid also for the conditions of a continuous gas formation and continuously changing gas pressure and the space occupied by them. Bearing in mind that

$$RT_1 = f,$$

we get:

$$\frac{f}{\theta'}\omega\psi - \frac{p(W_\psi + s\ell)}{\theta'} = \frac{\varphi m v^2}{2}. \qquad (52)$$

This is the fundamental equation of pyrodynamics depicting the relationship between ψ, p, v and ℓ. Actually, it is the equation of energy transformation by the equation of energy balance.

The left part of the equation depicts the change of internal energy $\omega\psi$ kg of the powder gases when their temperature is lowered from T_1^o to T^o, if the assumed mean values of heat capacity are c_w' and θ'. The right part of the equation represents the total external work done by the powder gases at the given instant due to the change of their thermal condition.

All the terms of the equation are expressed in units of work (kg · dm). This equation is called at times the "equivalence equation." The value of θ' is usually transposed to the right side, and the equation is solved for the second term:

$$p(W_\psi + s\ell) = f\omega\psi - \frac{\theta}{2}\varphi m v^2$$

or, replacing W_ψ by $s\ell_\psi$, we will have:

$$ps(\ell_\psi + \ell) = f\omega\psi - \frac{\theta}{2}\varphi m v^2. \qquad (53)$$

This equation is also known as the Resál equation, first developed by the author in 1864. Hereafter we shall consider the value of θ in the balance equation and in the fundamental equation as a value corresponding to the mean value of $T_{cp} = \frac{T_1 + T_Д}{2}$, but without its prime index. The subject equation contains the following variables characterizing the elements of burning of powder and of the projectile's motion: the burned portion of the charge ψ, the gas pressure p, the length of projectile's travel ℓ, and its velocity v.

The length ℓ_ψ of the free space in the chamber at a given instant is a function of ψ. Actually, ψ is the independent variable, because the pressure imparting motion to the charge-projectile-barrel system is obtained only as a result of the burning of the powder and of gas formation. Nevertheless, all the variables are interconnected and affect one another. In order to establish the relation between four variables, additional equations must be had.

These additional equations are represented in one form or another by the aforementioned relations for the burning law and by the equations of motion of the first and second forms.

If pressure p is determined from equation (52), we will have:

$$p = \frac{f\omega\psi - \frac{\theta}{2}\varphi mc^2}{W_\psi + s\ell}.$$

In pyrostatics we had the following expression for depicting pressure at a given instant:

$$p = \frac{f\omega\psi}{W_\psi}.$$

Inasmuch as in the numerator of the first formula a value is substracted from $f\omega\psi$ proportional to the work $\frac{\varphi mv^2}{2}$, and in the denominator the volume of the bore corresponding to the distance ℓ traversed by the projectile is added to the free space of the chamber, it becomes entirely clear that at the "same loading conditions" the pressure in the barrel, while the projectile is in motion and while the work is performed by the gases, will be smaller than when the powder is burned at constant volume.

CHAPTER 3 - INVESTIGATION OF THE FUNDAMENTAL RELATIONS

1. THE BASIC ENERGY CHARACTERISTICS

The energy equilibrium equation is valid with regard to both the first and the second period when the charge is already fully burned, when $\psi = 1$ and when the gases expand adiabatically. In such a case only the two variables T and v enter into the equation.

$$\frac{f\omega}{\theta} - \frac{RT\omega}{\theta} = \frac{\varphi mv^2}{2}.$$

Inasmuch as $f = RT_1$,

$$\frac{\varphi m v^2}{2} = \frac{f\omega}{\theta}\left(1 - \frac{T}{T_1}\right). \tag{54}$$

The left side of the equation represents the total exterior work done by the powder gases when a shot is fired; it increases with the decrease of temperature T and would have attained a maximum value, were it possible to cool the powder gases at firing to T = 0, - a condition impossible in actual practice because it would correspond to an efficiency equal to unity.

Nevertheless, if we assume in equation (54) T = 0, we will get

$$\frac{\varphi m v_{\Pi p}^2}{2} = \frac{f\omega}{\theta}, \tag{55}$$

i.e., the maximum amount of work performed by ω kg of powder gases if all the energy confined in them were utilized, i.e., if the gases were cooled to absolute zero.

$\frac{f\omega}{\theta}$ may be called the "full supply of energy" confined in ω kg of powder, and the velocity of the projectile $v_{\Pi p}$ (= v_{limit}) corresponding to the full utilization of energy - the limiting or maximum projectile velocity.

The full supply of energy of one kg of powder gas will be expressed by the formula

$$\Pi = \frac{f}{\theta}.$$

This value is called at times the powder "potential." Although the above limiting expression for Π has a theoretical meaning only, because in practice the gases cannot be cooled to absolute zero when a shot is fired, nevertheless it shows that the working capacity of powder gases can be increased either by increasing the force (energy) $f = \frac{p_a w_1}{273} T_1$, or by decreasing the value of $\theta = \frac{c_p}{c_w} - 1$.

The energy of the powder can be increased by increasing the specific volume of the powder gases w_1 (under normal conditions), or by elevating the burning temperature of the powder T_1.

As was shown above, θ depends on the composition and temperature of the gases: it decreases in value with increase of temperature and increases when the latter is decreased.

Hence a powder with a higher burning temperature will possess a greater supply of work not only because of energy f, but also because of the smaller value of θ.

Inasmuch as the gas temperature drops from T_1 to $T_\text{д}$ when a shot is fired (which corresponds to the projectile's passing through the muzzle face), the value of θ changes. However, this change is quite small and is usually considered to be a constant equal to the mean value of the given temperature interval. The value of θ can be found from the following formula:

$$\theta = \frac{c_p - c_w}{c_w} = \frac{A_1 - A_2}{A_2 + B_1 T} = \frac{1}{A + BT},$$

where

$$A = \frac{A_2}{A_1 - A_2} \quad \text{and } B = \frac{B_1}{A_1 - A_2}.$$

The average value of $\bar{\theta}$ can be found by means of the following formula:

$$\bar{\theta} = \frac{1}{(T - T_1)} \int_{T_1}^{T} \frac{dT}{A + BT} = \frac{A_1 - A_2}{T - T_1} \int_{T_1}^{T} \frac{dT}{A_2 + B_1 T} =$$

$$= \frac{2.303(A_1 - A_2) \log \dfrac{A_2 + B_1 T_1}{A_2 + B_1 T}}{B_1 (T_1 - T)}.$$

The variation of $\bar{\theta}$ for pyroxylin powder with temperature is given in table 20 [17].

Table 20

$\frac{T}{T_1}$	1	0.90	0.80	0.70	0.60	0.50	0.10
$T^{o}K$	2700	2430	2160	1890	1620	1350	270
$\bar{\theta}$	0.185	0.190	0.196	0.202	0.208	0.215	0.252

Inasmuch as $\frac{T_A}{T_1}$ is usually ≈ 0.70, θ approaches the value of 0.2. In most methods used for solving the fundamental problem of pyrodynamics the value of $\bar{\theta}$ is considered to be equal to 0.2. Theoretically it would have been correct to use different values of θ for the first and second periods: a smaller value for the first period while the gases have undergone little cooling, and a higher value for the second period at which time the gases had undergone a greater amount of cooling.

It should be noted that the values of the coefficients A_1, B, A_2 vary considerably with different authors, and this discrepancy may affect the value of T_1 as well as the value of θ. Furthermore, the heat capacity values are expressed by more complex relationships than a linear one.

According to the latest data, these relationships deviate from a linear one in the low temperature range; in the range of 3000° to 1500-2000° the relationship C_w, t approaches a linear one even according to these data(*).

Solving equation (55) with respect to $v_{\Pi p}$, we shall find an expression for the so-called "maximum or limiting projectile velocity":

$$v_{\Pi p} = \sqrt{\frac{2}{\varphi} \frac{f}{\theta} \frac{w}{m}} = \sqrt{\frac{2g}{\varphi} \frac{f}{\theta} \frac{w}{q}}. \tag{56}$$

As was mentioned above, the maximum velocity of the projectile corresponds to the full utilization of the energy and an efficiency equal to unity. Although this value cannot be attained in practice, it enters as a factor into the formulas for the projectile velocity v of both the first and the second periods, and the true projectile velocity usually increases with increase of $v_{\Pi p}$. An analysis of formula (56) will show that the increase of $v_{\Pi p}$ depends on the supply of powder energy f/θ and on the relative weight of the charge w/q with respect to the weight of the projectile q: it decreases with the increase of φ. Although the concept of maximum velocity can be obtained by assuming that $T = 0$ in the energy equilibrium equation,

(*) A.M. Litvin, "TEKHNICHESKAYA TERMODINAMIKA" (Technical Thermodynamics), 1947.

nevertheless, in order to apply the value of $v_{\Pi p}$ to actual practice, the value of θ must be taken as an average value in the temperature range of T_1, ..., $T_Д$ rather than in the T_1, ..., 0 range, because in computing the true projectile velocity which is proportional to $v_{\Pi p}$, the gas temperature in the first and second periods does not drop below $T_Д$. Thus the concept of maximum velocity is a conditional one, and it would be more appropriate to call it the "practical value of the maximum (or limiting) velocity."

Later on, when attempting to determine the dependence of φ on w/q, it will be shown that the maximum velocity $v_{\Pi p}$ tends towards a definite value, rather than towards infinity, even when the ratio of w/q is increased indefinitely.

The term "efficiency" signifies the ratio of the useful work done by the powder gases to the full supply of energy stored in a given powder charge.

The useful work performed by ω kg of gas is measured by the kinetic energy acquired by the projectile at the instant its base passes the muzzle face $\left(\frac{mv_Д^2}{2}\right.$, where $v_Д$ is the muzzle velocity of the projectile).

Denoting the efficiency by $r_Д$, we get:

$$r_Д = \frac{\frac{mv_Д^2}{2}}{\frac{f\omega}{\theta}} = \frac{\theta m v_Д^2}{2f\omega}.$$

In the case of ordinary weapons the value of $r_Д$ varies between 0.20 and 0.33.

Certain authors incorporate into the efficiency expression the coefficient of the fictitious mass φ which takes into account the auxiliary work items. Thus

$$r'_{Д} = \frac{\varphi m v_{Д}^{2}/2}{f\omega/\theta} = \frac{v_{Д}^{2}}{v_{Пр}^{2}}.$$

Comparing it with formula (54), we will see that

$$r'_{Д} = \frac{T_1 - T}{T_1} = 1 - \frac{T}{T_1}.$$

But $\frac{T_1 - T}{T_1}$ is "the coefficient of the Carnot cycle performed by an ideal gas"(*), and would have represented the actual efficiency of the cycle in the absence of auxiliary or secondary work done by the gases and in the absence of parasitic resistances which the gases must overcome. Therefore, in order to correctly depict the efficiency of the powder in a weapon, φ should not be included in the efficiency expression.

The $r'_{Д}$ value is of great importance in the theory of ballistics developed in the USSR, because it takes into account the totality of the work done by the powder gases in the weapon.

In some textbooks the full amount of work is expressed by

(*) O.D. Khvolson, "KURS FIZIKI" (A Course in Physics), Vol. II, p. 451, 1919.

the magnitude $II' = EQ$. But $FQ \neq f/\theta$ because Q is the quantity of heat determined experimentally in a calorimetric bomb when the gas of the burned powder is cooled from the burning temperature down to $t = 15^{\circ}C$ (or $288^{\circ}K$). The magnitude $\frac{f}{\theta}$ is the work the gases would be capable of doing if cooled from the burning temperature of the powder T_1 to 0° rather than to $288^{\circ}K$.

Hence the relation between EQ and $\frac{f}{\theta}$ will be expressed by the formula:

$$EQ = \frac{f}{\theta}\left(1 - \frac{288}{T_1}\right),$$

i.e., EQ is about 10% smaller than $\frac{f}{\theta}$, because $T_1 = 2700\text{-}2800^{\circ}K$.

This condition must be taken into account when determining the efficiency. If the value of the latter is given, it is of importance to know whether same is taken with respect to $\frac{f\omega}{\theta}$ (in which case it will be smaller) or with respect to EQ (in which case the efficiency will be greater).

"The coefficient expressing the utilization of a unit of charge η_ω by weight" is expressed by the muzzle energy of the projectile $\frac{mv_\text{д}^2}{2}$ per unit weight of charge ω:

$$\eta_\omega = \frac{mv_\text{д}^2}{2\omega} \quad \text{kg} \cdot \text{dm/kg}.$$

This value for specific gun systems approaches a constant, and depends mainly on the relative length of the gun, the powder thickness and the point at which it is burned (burning location).

For short-barreled, medium-caliber guns, $\eta_\omega = 1{,}200{,}000\text{-}1{,}400{,}000$ kg $\cdot$ dm/kg or 120-140 ton $\cdot$ m/kg; for small arms, $\eta_\omega = 100\text{-}110$ ton $\cdot$ m/kg.

For fully charged howitzers $\eta_\omega = 150$-160 ton · m/kg; η_ω decreased with the decrease of the charge. As the muzzle velocity of high-power artillery units is increased, the relative weight of the charge ω/q must increase also and with it the relative work necessary for displacing the charge itself (gases and powder); as a result, the relative useful work done in such guns becomes decreased and the value of η_ω drops to 90 ton · m/kg and lower.

The value of η_ω can be used for the approximate computation of the weight of charge ω necessary for imparting a given muzzle velocity $v_Д$ to a projectile of a given weight (mass) q:

$$\omega = \frac{mv^2_Д}{2} : \eta_\omega$$

η_ω and $r_Д$ are linked by the simple relation:

$$r_Д = \eta_\omega : \frac{f}{\theta}.$$

In certain applications of interior ballistics of great importance is the ratio between the mean pressure p_{cp} at a given point of the projectile's travel and the maximum pressure p_m in the bore of the barrel ($\eta = \frac{p_{cp}}{p_m}$).

The average pressure during the period of time it takes the projectile to move from $l = 0$ to $l = l_Д$ is

$$\eta_Д = \frac{p_{cp_Д}}{p_m}.$$

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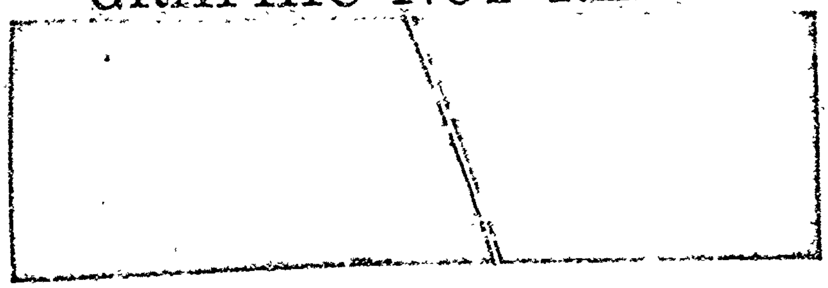

Fig. 88

Left: mean pressure when shot is fired.
Right: mean pressure as the characteristic of progressive burning of powder.

The mean pressure at a given point of the projectile's travel is that pressure which develops the same amount of work as a variable pressure, starting from p_0, passing through the maximum point and then decreasing in value.

Since the work done by the gases is depicted by the area $s\int_0^{\ell} p\,d\ell$, we can find p_{cp} from the condition

$$s p_{cp} \ell = s \int_0^{\ell} p\,d\ell.$$

In other words p_{cp} is the height of a rectangle whose area, when the base is ℓ, equals the area bounded by the pressure curve.

This is clarified by fig. 88,a; it also shows that as the length of bore travel ℓ is increased, p_{cp} decreases and has a minimum value when the projectile traverses the distance ℓ_{χ}, i.e., at the instant the projectile leaves the gun barrel.

If after passing point p_m (fig. 88,b) one pressure curve 1 proceeds above the other curve 2, this condition indicates that the mean pressure and the ratio $\frac{p_{cp}}{p_m}$ for the first curve are likewise greater than for the second curve.

And inasmuch as curve 1 points at more progressive burning than curve 2, the coefficient $\eta_Д$ also serves to depict the progressivity of burning: the greater the value of $\eta_Д$, the more progressive is the burning of the powder in the bore of the barrel.

Inasmuch as the characteristic $\eta_Д$ of the progressivity of burning must be known under certain conditions of firing, and only p_m and $v_Д$ can be determined by test when s, $\ell_Д$, q and ω are known, whereas the change of pressure p with reference to ℓ is often unknown, p_{cp} and then $\eta_Д$ are computed on the basis of the following considerations.

It is known that the work done by gases along the path $\ell_Д$ equals

$$\frac{\varphi m v_K^2}{2} = s\int_0^{\ell_Д} p\,d\ell;$$

on the other hand,

$$s\int_0^{\ell_Д} p\,d\ell = s p_{cpД}\ell_Д.$$

Therefore

$$s p_{cp.Д}\ell_Д = \frac{\varphi m v_Д^2}{2},$$

whence

$$p_{cp.\text{д}} = \frac{\varphi m v_{\text{д}}^2}{2s\ell_{\text{д}}}$$

and

$$\eta_{\text{д}} = \frac{p_{cp.\text{д}}}{p_m} = \frac{\varphi m v_{\text{д}}^2}{2s\ell_{\text{д}} p_m} = \frac{\varphi m v_{\text{д}}^2}{2W_{\text{д}} p_m}.$$

Inasmuch as the denominator in the expression for $\eta_{\text{д}}$ includes the swept volume of the bore $W_{\text{д}}$, $\eta_{\text{д}}$ is often called the "utilization coefficient of the swept volume of the bore."

Thus, in order to determine $\eta_{\text{д}}$ when a gun is fired, it is sufficient to know: the muzzle velocity of the projectile $v_{\text{д}}$, the maximum gas pressure p_m, the weight of the projectile, the cross-sectional area of the bore s, and the full length of travel of the projectile $\ell_{\text{д}}$ through the bore of the gun.

For cannons, the value of $\eta_{\text{д}}$ varies within the limits of 0.40 and 0.65.

The $\eta_{\text{д}}$ ratio can also be interpreted in a different manner. If we divide each term of the equation by $s\ell_{\text{д}} p_m$,

$$\eta_{\text{д}} = \frac{\varphi m v_{\text{д}}^2}{2s\ell_{\text{д}} p_m} = \frac{s\int_0^{\ell_{\text{д}}} p\,d\ell}{s p_m \ell_{\text{д}}} = \frac{\int_0^{\ell_{\text{д}}} p\,d\ell}{p_m \ell_{\text{д}}},$$

where the right part represents the ratio of the area bounded by the true pressure curve and the x-axis along the path $\ell_{\text{д}}$ to the area of a rectangle of height p_m and base $\ell_{\text{д}}$. This ratio will thus show the portion of the actual work done by the gases compared with the work done under ideal conditions if the pressure along the entire path $\ell_Д$ were equal to the maximum pressure p_m (fig. 89). For this reason $\eta_Д$ is often called the "coefficient of area closure on the indicator p, ℓ diagram."

The following characteristic can be introduced into the characteristic depicting the utilization of the entire barrel space, including the powder chamber space:

$$R_Д = \frac{\varphi m v_Д^2}{2s(\ell_0 + \ell_Д)p_m} = \frac{\varphi m v_Д^2}{2W_{KH}p_m},$$

which may be called the "coefficient of ballistic utilization of the entire bore space."

It can be easily seen that

$$R_Д = \eta_Д \frac{W_Д}{W_{KH}} = \eta_Д \frac{\ell_Д}{\ell_0 + \ell_Д} < \eta_Д.$$

Fig. 89 - Utilization of the Swept Volume of the Bore.

Fig. 90 - Utilization of the Entire Bore Space.

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Graphically R_{Ω} determines the ratio between the area bounded by the true pressure curve and the area of a rectangle of height p_m and base $\ell_0 + \ell_{\Omega}$ or $W_0 + W_{\Omega}$ (fig. 90).

2. THE DEPENDENCE OF PRESSURE CHANGE OF POWDER GASES IN THE GUN BARREL ON THE CONDITIONS OF LOADING

Using the pressure formula from the fundamental equation of pyrodynamics (53)

$$p = \frac{f\frac{\omega}{s}\psi - \frac{\theta}{2}\frac{\varphi m}{s}v^2}{\ell_\psi + \ell},$$

let us investigate the change of pressure with relation to time and the path traversed by the projectile. To do so, we shall find the derivatives

$$\frac{dp}{dt} \text{ and } \frac{dp}{d\ell} = \frac{dpdt}{dtd\ell} = \frac{1}{v}\frac{dp}{dt}.$$

Differentiating p with respect to t, we get

$$\frac{dp}{dt} = \frac{1}{(\ell_\psi + \ell)}\left[\frac{f\omega d\psi}{sdt} - \frac{\theta\varphi m}{s}v\frac{dv}{dt} - p\left(\frac{d\ell_\psi}{dt} + \frac{d\ell}{dt}\right)\right].$$

Bearing in mind that

$$\frac{d\psi}{dt} = \frac{S_1}{\Lambda_1}\frac{S}{S_1}u_1 p = \frac{\varkappa}{I_K}\sigma p = \Gamma p;$$

$$\frac{\varphi m}{s}\frac{dv}{dt} = p;$$

$$\frac{d\ell}{dt} = v;$$

$$\frac{d\ell_\psi}{dt} = \frac{d(\ell_\Delta - a\psi)}{dt} = -a\frac{d\psi}{dt} = -a\frac{\varkappa}{I_K}\sigma p = -a\Gamma p =$$

$$= -\frac{\omega}{s}\frac{\varkappa}{I_K}\frac{\sigma}{\delta_1}p;$$

$$\text{where } a = \frac{\omega}{s}\left(\frac{1}{\Delta} - \frac{1}{\delta}\right) = \frac{\omega}{s}\frac{1}{\delta_1},$$

and substituting them in the dp/dt formula, we get

$$\frac{dp}{dt} = \frac{p}{(\ell_\psi + \ell)}\left[\frac{f\omega}{s}\frac{\varkappa}{I_K}\sigma - \theta v - \left(v - a\frac{\varkappa}{I_K}\sigma p\right)\right] =$$

$$= \frac{p}{(\ell_\psi + \ell)}\left[\frac{f\omega}{s}\frac{\varkappa}{e_1}\sigma u_1\left(1 + \frac{1}{\delta_1}\frac{p}{f}\right) - v(1 + \theta)\right]. \quad (57)$$

This formula shows that the nature of pressure increase as a function of time depends on a large number of factors of varying influence.

At the start of motion when the rotating band is forced into the rifling grooves $p = p_0$, $\ell = 0$, $v = 0$, $\ell_\psi = \ell_{\psi_0}$, and formula (57) takes on the form:

$$\left(\frac{dp}{dt}\right)_0 = p_0\frac{f\omega}{s\ell_{\psi_0}}\frac{\varkappa\sigma_0}{I_K}\left(1 + \frac{1}{\delta_1}\frac{p_0}{f}\right) = p_0\frac{f\omega}{s\ell_{\psi_0}}\frac{\varkappa}{e_1}u_1\sigma_0\left(1 + \frac{1}{\delta_1}\frac{p_0}{f}\right) =$$

$$= p_0\frac{f\omega\Gamma_0}{s\ell_{\psi_0}}\left(1 + \frac{1}{\delta_1}\frac{p_0}{f}\right). \quad (58)$$

Therefore the rate of pressure increase at the start of motion is proportional to the wedging pressure p_0, the powder energy f, the weight of the charge ω, the exposed area of the powder $\frac{S_1}{\Lambda_1} = \frac{\varkappa}{e_1}$, the rate of powder burning at $p = 1$, i.e., u_1, and inversely proportional to the free space of the powder chamber sl_{ψ_0} at the instant wedging occurs. Inasmuch as the exposed area of the powder is inversely proportional to the thickness of the powder, $\left(\frac{dp}{dt}\right)_0$ is likewise inversely proportional to the powder thickness e_1. The first term in parentheses in formula (57) also depends on these magnitudes, which term expresses the intensity of energy developed by the powder gases.

If we replace $\frac{\varkappa}{e_1}u_1\sigma_0$ by $\frac{S_1}{\Lambda_1}u_1\sigma_0 = \Gamma_0$, we will obtain the additional condition where $\left(\frac{dp}{dt}\right)_0$ is proportional to Γ_0 for $\psi = \psi_0$; and since we had seen in our analysis of the physical law of burning that the outer layers of pyroxylin powders have an accelerated rate of burning (Γ develops ballooning), the pressure increase in this case will also be more intense than in the case of uniform burning u_1 assumed in the theoretical law.

Hence, all other conditions being equal, the pressure curve p, t in the case of the physical law of burning must proceed in the diagram above the corresponding p, t curve representing the theoretical law.

Formula (57) gives the tangent of the angle of inclination of the pressure curve as a function of time. At the start of motion the tangent of the angle has a specific limiting value depending on certain loading conditions (fig. 91,a); it becomes zero only when $p_0 = 0$. In this case the pressure curve as a function of time is tangent to the x-axis (fig. 91,b). This condition is not encountered in actual practice.

The nature of pressure increase as a function of path ℓ is expressed by the following formula:

$$\frac{dp}{d\ell} = \frac{1}{v}\frac{dp}{dt} = \frac{p}{(\ell_\psi + \ell)}\left[\frac{f\omega}{s}\frac{\varkappa}{I_k}\frac{\sigma}{v}\left(1 + \frac{p}{f\delta_1}\right) - (1 + \theta)\right]. \qquad (59)$$

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Fig. 91

Left: p, t curve at $p_0 > 0$; right: p, t curve at $p_0 = 0$.

At the start of motion $p = p_0$, $\ell = 0$, $v = 0$, $\sigma = \sigma_0$, $\ell_\psi = \ell_{\psi 0}$, and inasmuch as the first term in parenthesis is reduced to infinity,

$$\left(\frac{dp}{d\ell}\right)_0 = \infty .$$

Therefore, the ordinate is tangent to the p, ℓ curve at the start of motion (fig. 92).

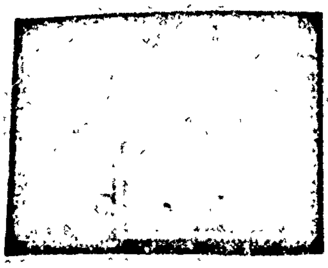

Fig. 92 - p, ℓ curve; the p-axis is tangent to the p, ℓ curve at the start of motion.

In order to obtain maximum pressure, $\frac{dp}{dt}$ or $\frac{dp}{d\ell}$ must be reduced to zero. Hence the condition at which p_m is obtained has the form:

$$f \frac{\omega}{s} \frac{\varkappa}{I_K} \sigma_m \left(1 + \frac{1}{\delta_1} \frac{p_m}{f}\right) - v_m(1 + \theta) = 0,$$

where the m index indicates that the given magnitude corresponds to p_m or

$$f \frac{\omega}{s} \Gamma_m \left(1 + \frac{1}{\delta_1} \frac{p_m}{f}\right) = (1 + \theta) v_m,$$

where

$$\Gamma_m = \frac{\varkappa}{I_K} \sigma_m = \frac{S_1}{\Lambda_1} u_1 \sigma_m.$$

If the requirements are such that the pressure must remain constant for a certain period of time after attaining a maximum value, a condition is obtained which must be satisfied by a change in surface area σ or by a change in the burning rate u_1:

$$f \omega \frac{\varkappa}{e_1} \sigma u_1 \left(1 + \frac{1}{\delta_1} \frac{p_m}{f}\right) = (1 + \theta) vs.$$

This condition can be formulated as follows.

In order to maintain the maximum pressure constant for a certain portion of the projectile's path in the bore, it is necessary that the surface area σ of the powder or the burning rate u_1 change in proportion to the projectile velocity v, or that the energy imparted by the powder gases at $p = 1(f\omega\Gamma)$ be proportional to the rate of volume change in the bore when the projectile is in motion

$$\left(sv = \frac{s\,d\ell}{dt} = \frac{dW}{dt}\right).$$

The pressure decreases after passing point p_m, the expression in parenthesis becomes negative, $\frac{dp}{dt} < 0$.

For the end of burning at $\psi = 1$, we get

$$\left(\frac{dp}{dt}\right)_K = \frac{p_K}{\ell_1 + \ell_K}\left\{\frac{f\omega}{s}\frac{\varkappa}{I_K}\sigma_K\left[1 + \left(\alpha - \frac{1}{\delta}\right)\frac{p_K}{f}\right] - (1 + \theta)v_K\right\}. \qquad (*)$$

Upon entering the second period the pressure equation takes on the form

$$p = \frac{f\omega}{s}\frac{1 - v^2/v^2_{np}}{\ell_1 + \ell}.$$

Differentiating, we get:

$$\frac{dp}{dt} = -(1 + \theta)\frac{v \cdot p}{\ell_1 + \ell}; \quad \frac{dp}{d\ell} = -(1 + \theta)\frac{p}{\ell_1 + \ell}$$

for the start of the second period

$$\left(\frac{dp}{dt}\right)_{(0)} = -(1 + \theta)\frac{v_K p_K}{\ell_1 + \ell_K}; \quad \left(\frac{dp}{d\ell}\right)_{(0)} = -(1 + \theta)\frac{p_K}{\ell_1 + \ell_K}.$$

Comparing this expression with (*), we note that at the instant of transition from the first period to the second, the derivatives dp/dt and $dp/d\ell$ undergo a drastic change (a jump), the pressure curve suffers a break, and the absolute value of the angle of inclination increases because of the disappearance of the first term in braces in expression(*).

Thereafter the angle of inclination of the p, t and p, ℓ curves becomes smaller, because p decreases and $\ell_1 + \ell$ increases; at the instant of the projectile's departure from the bore:

$$\left(\frac{dp}{dt}\right)_{\text{д}} = -(1 + \theta)\frac{p_{\text{д}} v_{\text{д}}}{\ell_1 + \ell_{\text{д}}}; \quad \left(\frac{dp}{d\ell}\right)_{\text{д}} = -(1 + \theta)\frac{p_{\text{д}}}{\ell_1 + \ell_{\text{д}}}.$$

3. THE EFFECT OF DIMENSIONS AND SHAPE OF POWDER ON THE GAS PRESSURE AND PROJECTILE VELOCITY CURVES

An analysis of formulas (57) and (59) will show that the pressure increase with respect to time and as a function of the path traversed by the projectile in the bore mainly depends on the term in brackets $f\omega\Gamma = f\omega\frac{\varkappa}{e_1}u_1\sigma$ which, for a given powder energy f, depends on the product $\omega\Gamma$, Γ being the intensity of gas formation at p = 1.

By changing the shape and dimensions of the powder, the magnitude $\frac{\varkappa\sigma}{e_1} = \frac{S_1}{\Lambda_1}\sigma$, which we shall denote by Σ, can be varied at will within wide limits.

The dependence of the change of $\varkappa$ and σ on the change of the powder grain shape is known from pyrostatics. The change of the pressure curve p, ℓ and of the projectile velocity curve v, ℓ with respect to the following can be illustrated by means of an example:

1) change of powder shape at the same powder thickness and the same charge ω, and

2) change of powder thickness at the same powder shape ($\varkappa$ = const, σ varies according to the same law) and at ω = const.

1. The effect of the grain shape when the thickness remains the same. We shall assume for the sake of simplicity that $e_1 = 1$, in which case $\Sigma = \varkappa\sigma$. At the start of burning at $z = 0$, $\sigma = 1$ and $\Sigma_0 = \varkappa$. The change of Σ corresponds to the change of σ. At the end of burning at $z = 1$

$$\Sigma_K = \varkappa\sigma_K = \varkappa(1 + 2\lambda + 3\mu).$$

By taking general formulas for five regressive powder shapes and using numerical data, we obtain table 21.

Table 21

	Shape of Powder	$\Sigma_0 = \varkappa$	σ_K	$\Sigma_K = \varkappa\sigma_K$	$\Sigma_0 = \varkappa$	σ_K	$\Sigma_K = \varkappa\sigma_K$
1	Tube	$1 + \beta$	$\frac{1 - \beta}{1 + \beta}$	$1 - \beta$	1.003	0.994	0.997
2	Strip	$1 + \alpha + \beta$	$\frac{(1 - \alpha)(1 - \beta)}{1 + \alpha + \beta}$	$(1 - \alpha)(1 - \beta)$	1.06	0.89	0.943
3	Plate	$1 + 2\beta$	$\frac{(1 - \beta)^2}{1 + 2\beta}$	$(1 - \beta)^2$	1.20	0.675	0.810
4	Slab	$2 + \beta$	0	0	~2.0	0	0
5	Cube	3	0	0	3	0	0

Upon plotting a graph of the change of Σ with respect to z, we will obtain the diagram shown in fig. 93.

GRAPHIC NOT REPRODUCIBLE

Fig. 93 - Change of Σ, z with Change of Powder Shape when ω and e_1 = const.

1) tube; 2) strip; 3) square plate; 4) rod; 5) cube.

The Σ, z diagram shows how the rate of gas formation changes when we cut up the same strip into square plates, then into strips and finally into cubes (see fig. 23).

The calculated action of these differently shaped powders (2, 4, 5) of the same thickness in the bore of a gun using the same charge ω by weight gives a diagram showing the changes of gas pressure as a function of the path traversed by the projectile in the bore (fig. 94).

These curves show that the strip generates a normal pressure p_m = 2380 kg/cm^2 and a muzzle velocity $v_{Д}$ = 590 m/sec, whereby the adiabatic curve of the second period attains the greatest height and the muzzle pressure is maximum.

Rod 4, whose exposed area is almost twice as great at the same powder thickness, generates almost double the pressure - p_m = 4600 kg/cm^2 and a considerably higher velocity $v_{Д}$ = 656 m/sec because of the large area of the p, l curve depicting the work done by the gases. The adiabatic curve of the second period drops sharply at the end of burning and in descending intersects the curve of the first period of strip powder and lies below the adiabatic curve of the strip powder.

The point of maximum pressure moves towards the point of the start of motion, as does the point representing the end of burning.

$\eta_K = \frac{\ell_K}{\ell_A} = 0.34$ for the strip, 0.18 for the rod, 0.128 for the cube. Inasmuch as the exposed area of the cube is three times as great, the cube generates a pressure of $p_m = 6200$ kg/cm^2 and a velocity $v_A = 681$ m/sec because of its still greater area $\int p d\ell$ than that of the rod (slab). The adiabatic curve of the second period lies still lower and gives the lowest muzzle pressure.

Thus, using the same powder thickness e_1 and the same charge by weight, we find that of the three powder shapes compared above the lowest pressure and the smallest velocity are produced by strip powder. The rod (slab) increases the pressure by almost 100%, whereas the velocity v_A increases only by 11%; the cube increases p_m almost 2.6 times and the velocity by 15.5%. In this case the regressive shape produces the maximum v_A, but at a pressure which is almost three times the normally allowable pressure. Hence, if the requirement, as is the case in actual practice, calls for the same pressure p_m with powders of different shapes, the area of the $\int p d\ell$ curves obtained with powders of greater regressivity will be smaller than the area obtained with strip powder, and the velocity v_A will be smaller. This represents the advantage of strip powder as the more regressive type.

For purposes of comparison, p, ℓ and v, ℓ curves are presented in the diagrams for the case (o), in which the powder is fully burned in the chamber before the projectile starts moving. This is the case of "instantaneous powder burning" in which the dimensions of the powder are infinitely small. This would obtain in practice when using dry powder-like pyroxylin.

In such a case the pressure curve starts from the maximum point, which pressure (11,700 kg/cm^2) is calculated according to the Noble formula. Thereafter the p, ℓ curve is entirely adiabatic in character, and the curves corresponding to various powder shapes arrange themselves in a proper manner (according to the established law) with respect to same. The obtained velocity v_A was found to be equal to 690 m/sec - i.e., the greatest, but the pressure p_{m0} was almost 5 times as great as p_m when strippowder was used (p_{m2} = 2380 kg/cm^2).

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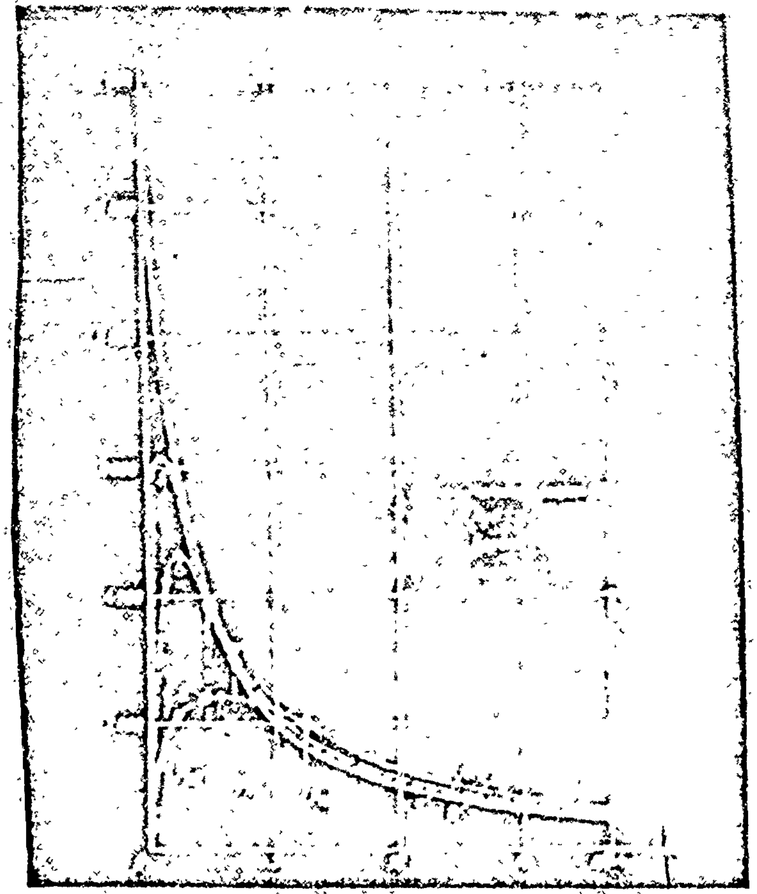

Fig. 94 - The Effect of the Grain Shape on the Pressure Curve Inside the Bore.

1) instantaneous burning (0); 2) cube (5); 3) slab (4); 4) strip (2); ordinate: g/cm².

2. The effect of web thickness on grains of the same shape. Grain shape - strip; thickness: $2e_1 = 1.5$, 2.0 and 2.5 mm.

$$\Sigma = \frac{\varkappa}{e_1}\sigma; \quad \Sigma_0 = \frac{\varkappa}{e_1} = \frac{1.06}{e_1};$$

$$\sigma_K = 0.889; \quad \Sigma_K = \frac{\varkappa}{e_1}\sigma_K = \frac{1.06 \cdot 0.889}{e_1} = \frac{0.943}{e_1}.$$

Table 22 - Magnitude

Thickness of Strip $2e_1$	$\Sigma_0 = \frac{1.06}{e_1}$	$\Sigma_K = \frac{0.943}{e_1}$	p_m, kg/cm^2	v_A m/sec
1.5	1.414	1.256	3540	632
2.0	1.060	0.943	2040	575
2.5	0.848	0.744	1450	486

The (Σ, z) diagram in fig. 95 shows that thinner powder generates a greater quantity of gas in a unit of time than thicker powder. The gas supply during burning can be regulated by changing the thickness of the powder.

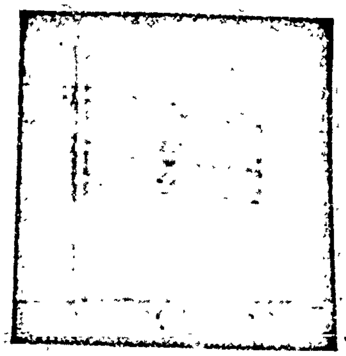

Fig. 95 - The Dependence of the Intensity of Gas Formation on the Powder Thickness.

Fig. 96 - The Effect of Powder Thickness on p,ℓ Curves in a Gun.

The calculated results of the action produced by these same powders in a gun using the same charge can be seen in the diagram of fig. 96; it shows that thin powder 1 generates a p_m which is 68% greater than normal pressure, whereas thicker powder 3 generates a pressure p_m which is 31% lower than normal pressure 2.

The muzzle velocities v_A are 632, 575 and 488 m/sec, respectively, where in the last case the thick powder does not succeed in getting

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fully burned inside the gun barrel, and hence the energy of a portion of the charge is not utilized at all.

The two examples cited above show the importance of the shape and dimensions of powder when firing a gun, and how the inflow of gases can be regulated and the desired rate of pressure change can be obtained by varying the thickness of the powder in combination with its shape.

If the requirement is such that p_m must not exceed a given value, the most regressive powder will be most suitable for the purpose, namely, tubular powders of the various shapes considered here, which approach the closest a powder with a constant burning area. It is possible however to obtain a progressive powder shape whose surface area increases with burning, and thus improve the efficiency of the weapon.

CHAPTER 4 - FORCES DEVELOPED IN A GUN WHILE THE PROJECTILE IS MOVING ACROSS THE RIFLING

1. THE RIFLED BORE. BASIC DESIGN DATA.

The bore of a gun barrel is rifled for the purpose of imparting a spinning motion to the projectile. The angle α formed by the grooves with the generatrix may be constant (rifling with a uniform twist) or variable, with the angle of twist increasing towards the muzzle (rifling with an increasing twist). The stability of the projectile's flight at a given velocity depends on the angle of twist α. The projectile's spinning motion is imparted by the pressure of the driving edges of the lands, which, in the case of a right-handed thread (clockwise rotation), is provided by the right-hand edge of the lands (fig. 97, a and b). These edges, upon encountering the rotating band of the projectile, develop a resistance N, which force is applied to the center of the protruding band and forces the projectile to rotate clockwise. A similar but a directly counteracting force N' is imparted by the projectile on the driving edge of the thread. Due to the elastic properties of the walls of the bore and the rotating band, radial forces Φ and frictional forces $\nu\Phi$ originate on the contacting surfaces (see fig. 98).

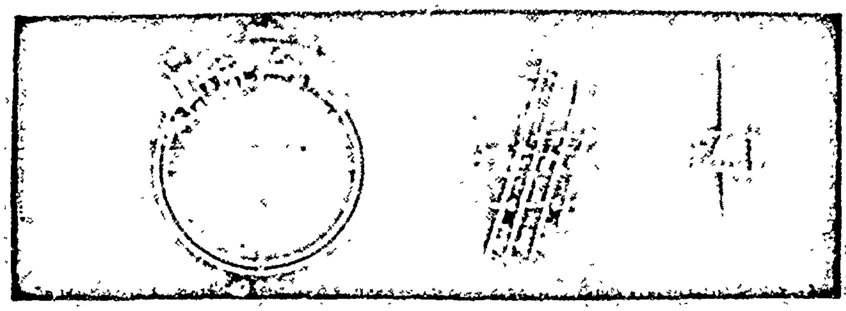

Fig. 97 - Rifling in a Gun Barrel

1) barrel; 2) projectile.

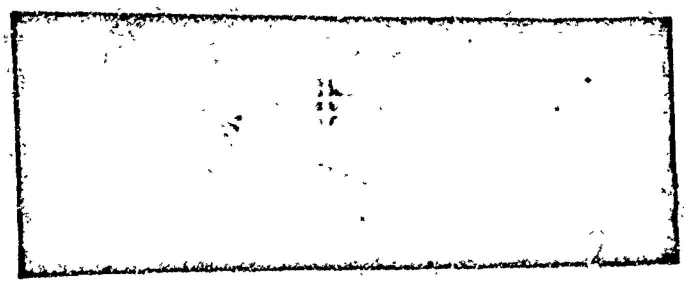

Fig. 98 - Forces Acting in the Rifling Grooves.

In addition to the twist angle α, the grooves are characterized by the land width a, the width b at the bottom of the groove, depth of grooves t_H and length (height) of driving band b_0. The force

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N is distributed over the area $b_0 t_H : \cos\alpha$, but inasmuch as at $\alpha = 8^\circ$, $\cos\alpha = 0.99 \approx 1$, the area $b_0 t_H$ is used for computing the stress in the groove and the rotating band. In artillery pieces the value of t_H is usually taken as $t_H \approx (0.01-0.02)d$, where d is the caliber of the bore or the diameter between the lands; $b_0 \approx 0.15d$.

For small-caliber weapons $t_H = (0.02-0.04)d$.

When the projectile moves through the bore, the gases exert a pressure on the base of the projectile as well as the ridges of the rotating band formed when the latter is forced into the grooves. Therefore the cross-sectional area s of the bore is greater than $\pi d^2/4$ and is calculated approximately by the formula $s = (0.80-0.83)d^2$ or by means of the more exact formula

$$s = \frac{\pi}{4}\left(\frac{a}{a+b}d^2 + \frac{b}{a+b}d'^2\right) = \frac{\pi}{4}\left(\frac{ad^2 + bd'^2}{a+b}\right) =$$

$$= \frac{\pi}{4}d^2\left[\frac{a + b\left(\frac{d'}{d}\right)^2}{a+b}\right].$$

The latter is obtained if we subdivide the whole area into pairs of sectors of diameters d and d' resting respectively upon arcs a and b. Of the two sectors subtended by the given angle, the sector resting on the land of the rifling occupies the portion $\frac{a}{a+b}$, and the one resting on the bottom of the thread occupies the portion $\frac{b}{a+b}$.

If we equate this area s to the area of an equidimensional circle, the diameter of the latter d_1 will represent the true caliber of the bore. It may be assumed that the force N spinning the projectile about its axis has a lever-arm d_1. For artillery pieces $d' \approx 1.02$; $d_1 \approx 1.01d$;

$$d_1 = \sqrt{\frac{ad^2 + bd'^2}{a + b}}.$$

The required number of grooves n is usually determined from the formula

$$n = (3 \div 3.5)d_{cm} \text{ or } n = 2d_{cm} + 8$$

rounded off to a multiple of four ($n = 4$ for small arms), in order to be able to cut the bore simultaneously with four cutters.

In addition to the angle of inclination α, the rifling is also characterized by the lead of the thread h, i.e., by the length of the generating line equivalent to a full turn of the thread (fig. 99):

$$h = \pi d \text{ cotan } \alpha.$$

The $\frac{h}{d}$ ratio is called the rifling twist or the lead of the rifling in calibers:

$$\eta = \frac{h}{d} = \frac{\pi}{\tan \alpha}.$$

h/d is usually given in round numbers (20, 25, 30...60) and the angle of inclination α is determined from them:

$$\alpha = \arctan \frac{\pi d}{h}.$$

Table 23

η	50	40	35	30	25	20
α	3°35'6"	4°30'	5°07'5"	5°58'7"	7°09'45"	8°56'

Rifling Equation. When the cylinder of the bore with an increasing rifling twist is developed on a plane, the rifling appears as a parabola (fig. 100), whose origin and angle of inclination $\alpha = 0$ lie below the actual thread on the extension of the thread curve.

The equation for this parabola is:

$$x^2 = ky \text{ or } y = \frac{x^2}{k},$$

$$\tan\alpha = \frac{dy}{dx} = \frac{2x}{k}; \quad \frac{d\tan\alpha}{dx} = \frac{2}{k} = \text{const},$$

i.e., the change of the angle of inclination versus the distance x remains constant.

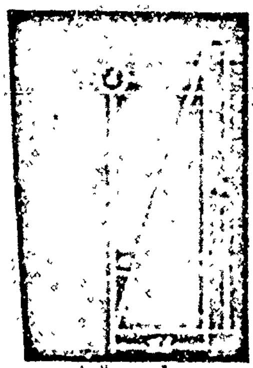

Fig. 99 - Uniform Twist Rifling Diagram

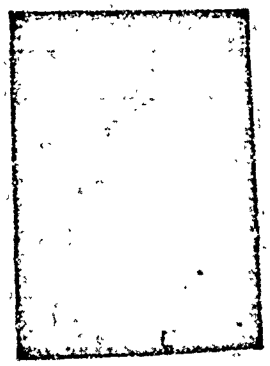

Fig. 100 - Increasing Twist Rifling Diagram

Since the angles of inclination α_1 and α_2 at the beginning and end of the rifled portion of the bore are known, the constant k can be determined. Indeed:

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$$\tan \alpha_1 = \frac{2c}{k}; \qquad \tan \alpha_2 = \frac{2(c + L_{Hp})}{k}, \tag{60}$$

whence

$$k = \frac{2L_{Hp}}{\tan \alpha_2 - \tan \alpha_1}, \tag{61}$$

and hence

$$\frac{d \tan \alpha}{dx} = \frac{\tan \alpha_2 - \tan \alpha_1}{L_{Hp}} = \text{const.}$$

The last expression enters into the formula expressing the pressure exerted on the driving edge.

Bearing in mind that $x = c + \ell_n$, where ℓ_n is the path traversed by the rotating band in the bore, and finding from (60) that

$$c = \frac{k}{2} \tan \alpha_1 = \frac{\tan \alpha_1}{(\tan \alpha_2 - \tan \alpha_1)} L_{Hp},$$

we will obtain the relation $\tan \alpha = \frac{2}{k} x$ from the path ℓ_n in the form

$$\tan \alpha = \frac{2(c + \ell_n)}{k} = \tan \alpha_1 + (\tan \alpha_2 - \tan \alpha_1) \frac{\ell_n}{L_{Hp}}. \tag{62}$$

ℓ_n varies from zero to $\ell_n = \ell_A - a$, where a is the distance between the base of the projectile and the forward edge of the rotating band.

We are presenting below the characteristic of several guns with relation to the weight of the recoiling parts Q_0, the weight of the projectile q, the weight of the charge ω and the type of rifling used(*) (Table 24).

2. THE RESISTANCE ENCOUNTERED BY THE ROTATING BAND WHEN FORCED INTO THE RIFLING GROOVES; PRESSURE TO OVERCOME THE INERTIA OF THE PROJECTILE.

When the projectile is properly seated, the forcing cone of the band must bear against the chamber cone and partly enter the tapered end of the rifling (fig. 101); this prevents the escape of the gases from the chamber. As the gas pressure increases, the band takes the grooves and fully enters the latter when the rear edge of the band a approaches the end of the groove taper (point b). The rotating band resistance is maximum at this point. The force Π_0 acting on the cross-sectional area of the bore s, i.e., $\frac{\Pi_0}{s} = p_0$ kg/cm^2, necessary to drive the band for its full length, is called the "pressure to overcome the inertia of the projectile."

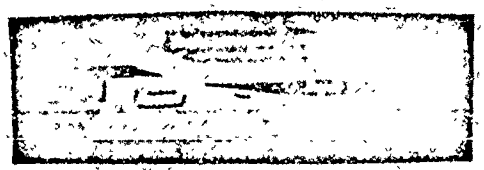

Fig. 101 - Rotating Band Entering the Rifling Grooves

1) connecting taper; 2) rotating band; 3) rifling.

(*) For rifling with an increasing twist, $\frac{h}{d}$ is given for the angle of inclination at the muzzle face of the bore.

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Upon entering the grooves to its full length, the copper band does not undergo further deformation, and the projectile continues to move with the ridges fully formed on its rotating band, following which the pressure undergoes a sudden drop.

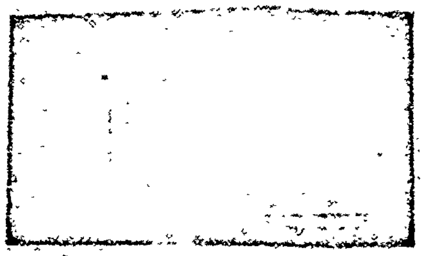

Fig. 102 - Change of Resistance as the Rotating Band is Forced into the Grooves.

a) kg/cm^2; b) path of projectile.

The resistance of the walls and grooves of the bore against the rotating band as the projectile moves through the barrel is determined by forcing the projectile through the bore by means of a mechanical or hydraulic press. This method was used by M.F. Rozenberg at the former Obukhov Plant in 1898 and by A.G. Maturin at the former Putilov Plant in 1899. However, this slow, static cold broaching operation usually produces high resistances between the rotating band and the bore due to the presence of the N. μN, Φ and $\nu\Phi$ forces.

The diagram in fig. 102 shows the change in pressure $p = II/s$ when forcing the band into the grooves and moving the projectile through a 76-mm gun 1902 issue. This diagram was obtained in tests conducted by (KOSARTOP) in 1925. It shows that the pressure developed in the 76-mm gun 1902 issue by the gradual forcing of the band into the grooves increases from 150 to 250 kg/cm^2, and abruptly drops to 70

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kg/cm^2 after the band is fully wedged in the grooves, following which it slowly decreases to 30 kg/cm^2 at the muzzle face of the gun.

Under actual firing conditions, the walls of the barrel near the rotating band undergo elastic deformation under gas pressure p, whereby the walls are displaced or stretched from position a to position a' (fig. 103); this deformation is transmitted for a certain distance forward and weakens the action of forces ϕ and $\nu\phi$.

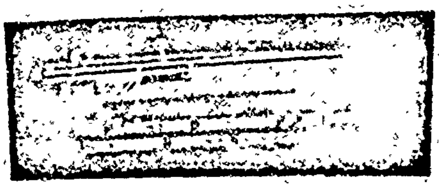

Fig. 103 - The Action of Pressure p in Displacing the Rotating Band of the Projectile

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Table 24 - Gun Characteristics

...of ...lle	Weight of charge	Muzzle energy $E_1 = \frac{mv_\Delta^2}{2}$ ton-m	$\gamma = \frac{d}{h}$	Angle of twist	Depth of groove, mm	Width in mm of land	Width in mm of bottom of groove	Number of grooves	Length of rifled part L_{Hp} in mm
	0.365	45.85	25	7°9'45"	0.76	3.05	6.91	24	1060
	1.080	146.10	25	7°9'45"	0.76	2.10	5.38	32	2663
	2.79	393.20	25	7°9'45"	1.0	3.0	7.47	32	3419
	7.56	952.40	20	3°54'25" 8°55'37"	1.5	3.0	6.97	48	3591
	2.125	311.90	20	3°42'00" 8°55'37"	1.25	3.81	9.47	36	1809
	1.170	147.10	20	3°42'00" 8°56'	1.015	3.04	7.60	36	1260

Table 24 - Gun Characteri

Type of gun	Weight of recoiling parts	Weight of projectile	Weight of charge	Muzzle energy $E_1 = \frac{mv_0^2}{2}$ ton-m	$\eta = \frac{d}{h}$	Angle of twist
	kg					
76-mm mountain gun 1909 issue	287	6.20	0.365	45.85	25	7°9'4
76-mm cannon 1902/30 issue	570	6.20	1.080	146.10	25	7°9'4
107-mm gun 1910/30 issue	1300	17.18	2.79	393.20	25	7°9'4
152-mm gun 1910/34 issue	1650	43.56	7.56	952.40	20	3°54' 8°55'
152-mm howitzer 1909/30 issue	1435	40	2.125	311.90	20	3°42 8°55
122-mm howitzer 1910/30 issue	570	21.76	1.170	147.10	20	3°42 8°56

Table 24 - Gun Characteristics

ght rge	Muzzle energy $E_1 = \frac{mv_A^2}{2}$ ton-m	$\frac{d}{h}$	Angle of twist	Depth of groove, mm	Width in mm of land	Width in mm of bottom of groove	Number of grooves	Length of rifled part L_{Hp} in mm
65	45.85	25	7°9'45"	0.76	3.05	6.91	24	1060
80	146.10	25	7°9'45"	0.76	2.10	5.38	32	2663
9	393.20	25	7°9'45"	1.0	3.0	7.47	32	3419
6	952.40	20	3°54'25" 8°55'37"	1.5	3.0	6.97	48	3591
25	311.90	20	3°42'00" 8°55'37"	1.25	3.81	9.47	36	1809
70	147.10	20	3°42'00" 8°56'	1.015	3.04	7.60	36	1260

This was substantiated by the KOSARTOP tests. The pressure necessary for the translation of the projectile was determined by firing a shortened 76-mm cannon. By using reduced charges and by selecting such charges whereby a half of the number of projectiles fired would be ejected from the bore and the other remain in it, it was determined that a pressure of 150 kg/cm^2 was not sufficient to drive the band into the grooves and would only produce a hardly perceptible imprint of the grooves on the forward portion of the rotating band. At a pressure of 225 to 275 kg/cm^2 some of the projectiles remained in the bore and a number of them was ejected from the bore and dropped near the gun. Thus only a small additional pressure was sufficient to eject the projectile from the bore; the action of forces Φ and $\nu\Phi$ was negligible. In consequence, we shall disregard these forces in our future analysis.

Thus the pressure p_0 necessary to overcome the inertia of the projectile is equal in the given case to 250 kg/cm^2. This value may vary considerably depending on the rifling and band design. In compiling his tables, Prof. N.F. Drozdov used the value of p_0 = 300 kg/cm^2; certain authors use p_0 = 400 kg/cm^2 in their calculations.

Krantz uses different values for p_0 varying from 270 kg/cm^2 for a 76-mm cannon to 550 kg/cm^2 for rifles, in which the grooves are relatively deep and the entire bullet rather than just the rotating band is forced to make the rifling.

Special tests conducted by Asst. Prof. P.N. Shkvornikov indicate a pressure of p_0 = 300-400 kg/cm^2 for rifles.

This pressure p_0 will change if the diameter of the band d_0 or its profile is changed.

The value of p_0 usually specified for medium-caliber guns is

$$p_0 = 250\text{-}350 \text{ kg/cm}^2.$$

Gabeau offers the following expression for determining p_0:

$$p_0 = 4\nu U \frac{H}{d'} \cos\alpha \left[1 + \sin\alpha \frac{\sin\alpha + \nu\cos\alpha}{\cos\alpha - \nu\sin\alpha}\right],$$

where U - Elastic limit of the rotating copper band, attained by the band while being forced into the grooves;

$H(b_0)$ - diameter of rotating band (reduced);

d' - caliber - reduced;

α - angle of twist;

ν - coefficient of friction.

U is determined from the expression

$$U = 3000\left(\frac{\Sigma}{100}\right)^{0.6} + 550,$$

where

$$\frac{\Sigma}{100} = 0.1 + 1.1 \frac{D_0 - d'}{d' - d_{CH}};$$

D_0 - maximum diameter of rotating band,

d_{CH} - diameter of projectile body at the rotating band.

If we disregard the resistance present after the band is fully driven into the grooves, the motion of the projectile may be assumed to start at the instant the pressure attains the value p_0 produced by the partial burning of the charge ψ_0.

In order to obtain pressure p_0 at constant volume, it is necessary that a portion of charge ψ_0 determined from the following general pyrostatics equation be burned:

$$\psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f}{p_0} + \alpha - \frac{1}{\delta}}.$$

Thus the pressure to overcome the inertia of the projectile is indirectly accounted for by the magnitude ψ_0 of the portion of the charge burned by the time the projectile starts moving and by the corresponding relative part of the burned thickness z_0. The initial value $\left(\frac{dp}{dt}\right)_0$ also depends on the value of p_0; the higher the value of p_0, the greater $\left(\frac{dp}{dt}\right)_0$, the steeper the p, t curve at the start of the projectile's motion, and the higher the maximum pressure p_m.

3. FORCES DEVELOPED AT THE DRIVING EDGES WHEN THE PROJECTILE IS IN MOTION

The angle formed by the bore axis and the direction of the rifling grooves creates reaction forces between the driving edges of the rifling and the rotating band as the projectile moves through the bore. These forces N at each groove are directed perpendicularly to the surfaces of contact and create friction forces $\nu_1 N$ along the driving edge in a direction opposite to that of the projectile's motion.

These forces and their components (acting in the direction of the bore axis and a plane perpendicular to it) impart a spinning motion to the projectile and develop forces which counteract the translation of the projectile (force R).

In order to determine the reaction force N of the groove and the braking force R, let us imagine the bore surface developed a plane xy, with the x-axis parallel to the bore axis (fig. 104). Curve 00 depicts a groove with an increasing twist. Point A corresponds to the center of the driving edge, where $\frac{ps}{n}$, N and νN represent the forces acting on each driving edge, and n is the number of grooves.

We shall disregard the radial force Φ and the force of friction $\nu\Phi$.

Let us resolve forces N and νN into their components along the x and y axes:

$$N'' = N \sin \alpha; \quad N' = N \cos \alpha;$$

$$\nu N' = \nu N \cos \alpha; \quad \nu N'' = \nu N \sin \alpha.$$

According to the law of mechanics, we will have the following equation of motion:

1) The sum of the projections of forces along the x-axis equals $m \frac{d^2x}{dt^2}$,

2) The sum of the moments of rotating forces equals $I \frac{d^2\varphi}{dt^2} = I \frac{d\Omega}{dt}$,

where I is the moment of inertia of the projectile about the longitudinal axis;

Ω is the angular velocity of the projectile.

The moment of inertia of a projectile of mass m is

$$I = \Sigma \Delta m_i \cdot r_i^2 = \int r^2 \, dm,$$

where Δm - an element of the mass, located a distance r_i from the axis of rotation, can be represented as:

$$I = m\rho^2;$$

here ρ is the radius of gyration, which is determined as follows.

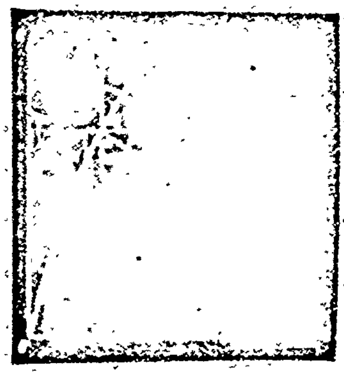

Fig. 104 - Forces Acting in the Rifling Grooves.

If the entire mass of the body rotated about the axis were concentrated in an infinitely thin cylindrical layer at a distance ρ from the axis, and the value of ρ is so chosen that the moment of inertia $m\rho^2$ is equal to the true moment of inertia I, then this distance from the axis of rotation would represent the radius of gyration.

Let us write the equation of rotary motion for the projectile:

$$nrN(\cos\alpha - \nu\sin\alpha) = I\,\frac{d\Omega}{dt}. \tag{63}$$

Inasmuch as

$$\Omega = \frac{v\tan\alpha}{r},$$

$$\frac{d\Omega}{dt} = \frac{1}{r}\left(\tan\alpha\,\frac{dv}{dt} + v\cdot\frac{\tan\alpha}{dx}\,\frac{dx}{dt}\right) = \frac{1}{r}\left(\tan\alpha\,\frac{dv}{dt} + v^2\,\frac{d\tan\alpha}{dx}\right);$$

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for rifling with an increasing twist

$$\frac{d \tan \alpha}{dx} = \frac{\tan \alpha_2 - \tan \alpha_1}{L_{\text{np}}} = k_\alpha = \text{const}$$

and for rifling with a uniform twist

$$k_\alpha = 0.$$

Upon substituting the expression for I and $\frac{d\Omega}{dt}$ in formula (63) and determining the value of N, we will get:

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 \frac{\left(\tan \alpha \cdot m \frac{dv}{dt} + k_\alpha m v^2\right)}{\cos \alpha - \nu \sin \alpha}. \tag{64}$$

In order to determine the value of $m \frac{dv}{dt}$ in parenthesis, we shall write the equation of translation:

$$p_{CH}s - nN(\sin \alpha + \nu \cos \alpha) = m \frac{dv}{dt}, \tag{65}$$

where p_{CH} is the gas pressure acting on the base of the projectile;

$$nN(\sin \alpha + \nu \cos \alpha) = R.$$

is the resisting force due to the reaction of n driving edges.

$$p_{CH}s - R = p_{CH}s \left(1 - \frac{R}{p_{CH}s}\right) = m \frac{dv}{dt},$$

The value of $\frac{R}{p_{CH}s}$ is small compared with unity, and

$$\frac{1}{1-\frac{R}{p_{CH}s}} \approx 1 + \frac{R}{p_{CH}s} = \varphi_1,$$

where φ_1 is a value slightly exceeding unity; this value will be determined with greater accuracy later on.

Therefore,

$$m\frac{dv}{dt} = \frac{p_{CH}s}{\varphi_1}.$$

Substituting this expression in formula (64), we get:

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 \frac{\tan\alpha\, sp_{CH} + \varphi_1 k_\alpha m v^2}{\varphi_1(\cos\alpha - \nu\sin\alpha)}.$$

The expression in the denominator closely approaches unity:

$$\varphi_1(\cos\alpha - \nu\sin\alpha) \approx 1.$$

We thus get the final expression for the reaction N of the groove if the rifling has an increasing twist:

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 (\tan\alpha\, sp_{CH} + \varphi_1 k_\alpha m v^2). \tag{66}$$

For grooves having a uniform twist $k_\alpha = 0$ and the force is

$$N = \frac{1}{n}\left(\frac{\rho}{r}\right)^2 \tan\alpha\, sp_{CH}. \tag{67}$$

The value of $\left(\frac{\rho}{r}\right)^2 = \lambda$ depends on the type of the projectile and varies between 0.48 for a bullet and 0.68 for a thin-walled highexplosive percussion shell.

For example:

	$\left(\frac{\rho}{r}\right)^2$
for a circular solid cylinder....................	0.50
for a solid bullet..............................	~0.48
for armor-piercing thick-walled shells..........	~0.56
for thin-walled percussion shells..............	0.64–0.68.

In order to compute the stress in the band metal, the force N must be referred to the contact area between the driving edges of the rifling and the band, i.e., to $b_0 t_H$, where t_H is the depth of the rifling and b_0 is the width of the rotating band.

Formula (67) shows that for rifling with a constant twist the pressure exerted by the band of the projectile on the driving edge of the rifling and, inversely, the pressure exerted by the driving edge on the band while the projectile is in motion, varies in proportion to the pressure exerted by the gases on the base of the projectile. Therefore, curve 1 representing the change of force N as a function of ℓ is similar to the pressure curve (fig. 105), and the band of the projectile and the driving edge are subjected to a maximum stress at the instant the pressure and the velocity of the projectile are at a maximum.

Fig. 105 - Effect Produced by Rifling on the Pressure N in the Rifling Grooves.

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Formula (66) indicates that by decreasing the initial groove angle α_1, the first term in parenthesis for the instant at which p_m is developed can be considerably decreased. Inasmuch as the velocity of the projectile at this instant is still small (as is the doubled kinetic energy of the projectile, mv^2), then at the instant of maximum developed pressure the value of N obtained according to formula (66) (in the case of an increasing twist) may be smaller than that obtained by formula (67) (for rifling with a uniform twist).

As the pressure continues to decrease, the $\tan\alpha$ increases, as does the second term $k_\alpha mv^2$. Hence, in the case of an increasing twist, the pressure acting on the driving edge varies more uniformly than in the case of rifling with a uniform twist. The force N may be varied considerably by changing the angles α_1 and α_2.

The curves in fig. 105 show the change of force N as a function of the path of the projectile for rifling grooves with α = const and for two riflings with a variable α:

1) $\alpha = 10^\circ$ = const;
2) $\alpha_1 = 5^\circ$, $\alpha_2 = 10^\circ$;
3) $\alpha_1 = 2^\circ$, $\alpha_2 = 10^\circ$.

If the rifling equation is known, the dependence of $\tan\alpha$ on the path of the projectile ℓ can be determined according to formula (62) as a means for calculating the change of N:

$$\tan\alpha = \tan\alpha_1 + (\tan\alpha_2 - \tan\alpha_1)\frac{\ell_n}{\ell_{Hp}} = \underline{\tan\alpha_1 + k_\alpha \ell_n};$$

and curves p, ℓ and v, ℓ can then be plotted and the values of α, p and v for the same values of ℓ substituted in formula (66). By substituting expression (62) in equation (66), we will get:

$$N = \frac{\lambda}{n}(\tan \alpha_1 sp + k_\alpha \ell_n sp + k_\alpha mv^2) = \frac{\lambda}{n} \tan \alpha_1 sp + \frac{\lambda}{n} k_\alpha (\ell sp + mv^2).$$

The first term represents that pressure N_{α_1} which would obtain in a rifling with a uniform twist whose angle $\alpha = \alpha_1$. This change is similar to the change of the pressure curve p, ℓ, i.e., it first increases and then decreases. The second term depends on ℓ, p and v, whereby, ℓ, p and v increase until the pressure attains a maximum value, following which ℓ and v continue to increase and p decreases. This formula makes it possible to analyze the influence of each variable p, v and ℓ on the pressure exerted on the driving edge.

The resistance offered by the rifling against the translatory motion of the projectile is

$$R = nN(\sin \alpha + \nu \cos \alpha) = nN \cos \alpha(\tan \alpha + \nu). \tag{68}$$

For rifling with a uniform twist (assuming $\cos \alpha \approx 1$):

$$R = \left(\frac{\rho}{r}\right)^2 (\tan^2 \alpha + \nu \tan \alpha) sp_{CH}, \tag{69}$$

i.e., R is proportional to the gas pressure on the base of the projectile.

The magnitude φ_1 introduced above, which takes into account the braking effect of the rifling grooves on the motion of the projectile, is also constant when the rifling twist is uniform:

$$\varphi_1 = 1 + \frac{R}{sp_{CH}} = 1 + \left(\frac{\rho}{r}\right)^2 (\tan^2 \alpha + \nu \tan \alpha).$$

The equation of translatory motion of the projectile can be written as follows:

$$sp_{CH} = \varphi_1 m \frac{dv}{dt}$$

Since $\varphi_1 > 1$, the resistance offered by the rifling is the same as if the mass of the projectile were increased. In the expression $\varphi_1 = 1 + \lambda \tan^2 \alpha + \lambda \nu \tan \alpha$ the value of the coefficient $\lambda \tan^2 \alpha$ varies between 0.0025 $\left(\frac{1}{4}\%\right)$ for a small pitch with a rifling pitch h = 45 calibers and 0.025 (2.5%) for a very steep pitch (h = 15 calibers).

For a medium angle of twist $\alpha = 6\text{-}7^{\circ}$ $\lambda \tan^2 \alpha \approx 0.01$ (1%).

The value of the coefficient $\lambda \nu \tan \alpha$ depends on both the angle α and the coefficient of friction ν which is usually taken between 0.16 and 0.20; on the average $\lambda \nu \tan \alpha \approx 0.01$ (1%).

Investigations made during the past few years have shown that ν decreases as the velocity of the projectile increases; at $v \approx 200$ m/sec $\nu \approx 0.10$, and at $v = 1000$ m/sec $\nu \approx 0.05$.

φ_1 s usually taken to be equal to $\varphi_1 = 1.02$.

For rifling with an increasing twist

$$R = \frac{\lambda}{n}(\tan \alpha \; sp_{CH} + \varphi_1 k_\alpha \; mv^2)(\tan \alpha + \nu),$$

and the magnitude

$$\varphi_1 = 1 + \frac{R}{sp_{CH}}$$

will no longer be a constant value.

4. THE WORK DONE IN OVERCOMING THE RESISTANCE R OFFERED BY THE RIFLING GROOVES.

In order to overcome the resistance R, the powder gases must do a certain amount of work.

For rifling with a uniform twist:

$$R = nN \cos\alpha(\tan\alpha + \nu) = \lambda(\tan^2\alpha + \nu\tan\alpha)sp_{CH}.$$

The work done in overcoming this resistance is

$$\int_0^t R d\ell = \lambda(\tan^2\alpha + \nu\tan\alpha)s\int_0^t p_{CH}d\ell,$$

but

$$s\int_0^t p_{CH}d\ell = \frac{mv^2}{2}.$$

Therefore,

$$\int_0^R R d\ell = \lambda\tan^2\alpha\frac{mv^2}{2} + \lambda\nu\tan\alpha\frac{mv^2}{2}.$$

It can be shown that the first term represents the work done in imparting a spinning motion E_2 to the projectile, and the second term represents the work done in overcoming friction E_3.

Both types of work are proportional to $\frac{mv^2}{2} = E_1$ (to the work done in imparting translation to the projectile) and can be written as:

$$E_2 = \lambda \tan^2 \alpha \frac{mv^2}{2} = k_2 \frac{mv^2}{2},$$

where $k_2 = \lambda \tan^2 \alpha$;

$$E_3 = \lambda \nu \tan \alpha \frac{mv^2}{2} = k_3 \frac{mv^2}{2},$$

where $k_3 = \lambda \nu \tan \alpha$.

Comparing it with the expression for φ_1, we find that

$$\varphi_1 = 1 + k_2 + k_3.$$

For rifling with an increasing twist:

$$R = nN(\sin \alpha + \nu \cos \alpha) = \lambda (\tan \alpha s p_{CH} + \varphi_1 k_\alpha m v^2)(\sin \alpha + \nu \cos \alpha) =$$

$$= \lambda \cos \alpha (\tan \alpha s p_{CH} + \varphi_1 k_\alpha m v^2)(\tan \alpha + \nu).$$

Substituting therein the expressions

$$\tan \alpha = \tan \alpha_1 + k_\alpha \ell$$

and

$$\tan^2 \alpha = \tan^2 \alpha_1 \left[1 + 2n_\alpha \frac{\ell}{L_{np}} + n_\alpha^2 \left(\frac{\ell}{L_{np}}\right)^2\right],$$

where

$$n_\alpha = \left(\frac{\tan \alpha_2}{\tan \alpha_1} - 1\right),$$

we will get an expression for R as a function of the projectile's path ℓ and its velocity v, and using the numerical value of $\int_0^1 R d\ell$ we can determine the work done in overcoming the resistance offered by a rifling with an increasing twist.

CHAPTER 5 - DERIVATION OF FORMULAS FOR DETERMINING THE SECONDARY TYPES OF WORK INVOLVED

1. WORK DONE IN SPINNING THE PROJECTILE

The work done in imparting a spinning motion to the projectile is expressed by the formula

$$E_2 = \frac{I\Omega^2}{2},$$

where I = moment of inertia of the projectile about the axis of rotation ($I = m\rho^2$);

Ω = angular speed of rotation.

It was shown above that all four items of work under consideration are proportional to the basic work $E_1 = \frac{mv^2}{2}$, and the expression for E_2 can be reduced to the form:

$$E_2 = k_2 \frac{mv^2}{2},$$

from which we can determine the value of k_2. We shall substitute the linear speed v for the angular speed of the projectile Ω:

$$\Omega = \frac{v \tan \alpha}{r};$$

$$E_2 = \frac{m\rho^2 v^2 \tan^2 \alpha}{2r^2} = \left(\frac{\rho}{r}\right)^2 \tan^2 \alpha \frac{mv^2}{2} = k_2 \frac{mv^2}{2},$$

where

$$k_2 = \left(\frac{\rho}{r}\right)^2 \tan^2 \alpha.$$

The coefficient k_2 indicates the portion of the total work done in spinning the projectile. This magnitude depends on the design or type of the projectile $\left(\frac{\rho}{r}\right)^2$ and on the rifling of the bore - the angle of twist α.

2. THE WORK DONE IN OVERCOMING THE FRICTION IN THE RIFLING GROOVES

The component of the force of friction on the driving edge resisting the projectile's motion is expressed by:

$$n \nu N \cos \alpha.$$

The work done in overcoming this resistance is

$$E_3 = \int_0^{\ell} n\nu N \cos \alpha \frac{d\ell}{\cos \alpha},$$

because the path traversed along the rifling is $\ell/\cos \alpha$.

Substituting here the expression for N:

$$E_3 = \left(\frac{\rho}{r}\right)^2 \nu \tan \alpha \; s \int_0^t p_{CH} d\ell = \left(\frac{\rho}{r}\right)^2 \nu \tan \alpha \frac{mv^2}{2}.$$

we thus get

$$k_3 = \left(\frac{\rho}{r}\right)^2 \nu \tan \alpha,$$

i.e., the expression derived earlier.

The numerical values of k_2 and k_3 were likewise discussed earlier in the text.

3. THE WORK DONE IN DISPLACING THE CHARGE

In displacing the projectile through the bore, the powder gases move together with it, whereby the unburned portion of the charge may move likewise under the action of non-uniform pressures developed in the bore. A portion of the developed energy is thus spent on the displacement of certain portions of the charge and on imparting kinetic energy to them, which must be taken into consideration.

Inasmuch as no accurate data is available on the distribution of the gas mass and the unburned porti n of the charge in the initial air space, certain allowances must be resorted to when these factors are taken into account.

It is known that when a shot is fired wave motions may occur when the gases impinging on the base of the projectile rebound and encounter other gases flowing towards them, thus creating a localized pressure rise. Furthermore, in entering the narrower bore from the chamber, the gas stream becomes smaller in cross section, and this may also make it more difficult to express the law of motion in the form of analytic functions. As a result, it is necessary to resort to certain simplified expressions and allowances when determining the work done in the displacement of the gases.

This problem is presented as follows: an expression must be obtained for the kinetic energy of the portions of the charge moving with a variable speed, and an expression for linking this with the kinetic energy of the projectile.

In solving this problem, we shall make the following allowances:

1) The bore, including the powder chamber, has the same area,

equal to the cross-sectional area s.

2) At each position occupied by the projectile, the mass of the charge is distributed evenly throughout the entire space between the base of the projectile and the base of the bore.

3) The elements of the charge have a translatory motion only, and the velocities between its layers increase from zero at the base of the chamber to v at the base of the projectile according to the linear law.

4) The velocities of the particles at a given cross section are the same, and no friction exists between the particles of the charge and the walls of the bore.

The diagram in fig. 106 clarifies the above.

We shall designate:

v - velocity of the projectile;

v_ω - velocity of a charge element in a given layer;

$\mu = \omega/g$ - mass of charge;

Λ - distance between chamber base and base of projectile.

At the instant the projectile had traversed a distance ℓ, Λ is constant; it varies with time, whereas we are considering the condition of the charge masses in the initial air space at various distances x from the base of the chamber at every instant.

We shall separate an elementary layer of cross section s and height dx and designate its mass by $d\mu$. The layer moves with a velocity v_ω; its elementary kinetic energy will be expressed thus:

$$dE_4 = \frac{d\mu v_\omega^2}{2}.$$

In order to obtain an expression for the full kinetic energy, this expression must be integrated for ℓ between zero and Л. We will then find the kinetic energy of the charge, whose elements move in the initial air space according to a given law.

We have from the condition of uniform mass distribution:

$$\frac{d\mu}{\mu} = \frac{dx}{\text{Л}} \text{ or } d\mu = \frac{\mu}{\text{Л}}\, dx.$$

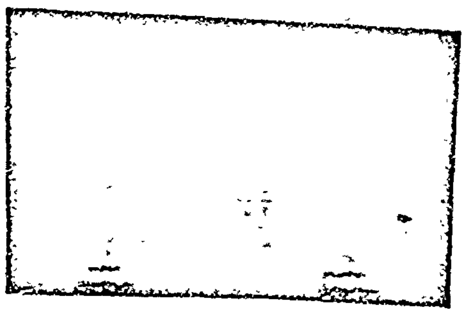

Fig. 106 - Distribution of Velocities of the Gas Layers Back of the Projectile

a) chamber base; b) projectile base.

From the condition that the velocity v_ω changes according to the linear law and from similar triangles:

$$v_\omega : v = x : \text{Л},$$

whence

$$v_\omega = \frac{v}{\text{Л}} x.$$

Upon substituting, we get:

$$dE_4 = \frac{v^2}{\text{Л}^2} \frac{x^2}{2} \cdot \frac{\mu}{\text{Л}}\, dx = \frac{\mu v^2}{2\text{Л}^3} x^2 dx.$$

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Integrating between the limits of zero and η, we get:

$$E_4 = \int_0^{\eta} \frac{v_\omega^2 d\mu}{2} = \int_0^{\eta} \frac{\mu v^2}{2\eta^3} x^2 dx$$

or

$$E_4 = \frac{\mu v^2}{2\eta^3} \int_0^{\eta} x^2 dx = \frac{\mu v^2 \eta^3}{2\eta^3 . 3} = \frac{1}{3} \frac{\mu v^2}{2} = \frac{1}{3} \frac{\mu}{m} \frac{mv^2}{2};$$

because

$$\frac{\mu}{m} = \frac{\omega}{q},$$

then

$$E_4 = \frac{1}{3} \frac{\omega}{q} \frac{mv^2}{2} = k_4 E_1.$$

Hence, in consequence of the allowances made, the work done in moving the gases and the charge is proportional to the basic work E_1. The coefficient $k_4 = \frac{1}{3} \frac{\omega}{q}$ varies considerably depending on the relative weight of the charge $\frac{\omega}{q}$. In low-power guns and howitzers $\frac{\omega}{q} \approx 0.10 \div 0.15$, in high-power guns it is about $0.30 \div 0.40$. Consequently,

$$k_4 = \frac{1}{3} \frac{\omega}{q} = 0.03 \div 0.13.$$

This coefficient was obtained under specific assumptions regarding the distribution of masses and velocities. In actuality this phenomenon is much more complex and permits certain allowances. For example, F.F. Lender, in assuming that the greater portion of the charge is concentrated nearer the chamber base and hence has a smaller velocity, obtained a coefficient $b_4 = \frac{1}{6}$; other authorities assume b equal to 1/4 or 3/11.

4. THE EFFECT OF WIDENING THE CHAMBER ON THE WORK DONE IN DISPLACING THE GASES

The motion of gases in the presence of a widened chamber represents a complex problem in gas dynamics which has not been solved to this day. We shall resort to a simpler and less accurate relation which takes into account the effect of a widened chamber on the k_4 coefficient.

The following allowances must be made in deriving this relation.

1) The gas mass is distributed uniformly in the initial air space, but only that part of the mass is in motion whose cross-sectional area s equals the cross-sectional area of the bore. The outer layers adjoining the chamber walls do not participate in this movement. As usual, the interval gas friction and the friction between the gases and the walls of the bore are disregarded.

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Fig. 107 - Motion of Gases in the Presence of a Widened Chamber.

2) The velocity of the gas layers participating in the motion

varies linearly from zero at the base of the chamber to that of the velocity of the projectile at its base.

By using this purely mechanical representation which does not take into account the gas dynamics relating to the compression of the gas stream, we obtain the diagram shown in fig. 107.

The weight of the gases ω' participating in this motion relative to the over-all weight of the charge ω is expressed by the following formula:

$$\frac{\omega'}{\omega} = \frac{s(l_{KM} + l)}{W_0 + sl} = \frac{l_{KM} + l}{l_0 + l} = \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1},$$

where $\chi = \frac{l_0}{l_{KM}}$ is the coefficient of expansion (widening) of the chamber and $\Lambda = \frac{l}{l_0}$.

$$\omega' = \omega \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}.$$

The formula for k_4 derived above is applicable to this gas mass:

$$k_4' = \frac{1}{3} \frac{\omega'}{q} = \frac{1}{3} \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1} \frac{\omega}{q} = b \frac{\omega}{q}$$

when $\chi = 1$, $k_4 = \frac{1}{3} \frac{\omega}{q}$, $b = \frac{1}{3}$.

As Λ changes with the movement of the projectile, the coefficient b varies from $b = \frac{1}{3} \frac{1}{\chi}$ at the start of motion when $\Lambda = 0$ and tends towards $b = \frac{1}{3} \frac{\omega}{q}$ as Λ increases.

Because Λ varies, when integrating the equation of interior ballistics, the value of b must be taken as the average value between 0 and $\Lambda_{\text{д}}$:

$$b_{cp} = \frac{1}{3}\left(\frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}\right)_{cp} = \frac{1}{3}\frac{1}{\Lambda}\int_0^{\Lambda} \frac{\Lambda + \frac{1}{\chi}}{\Lambda + 1}\, d\Lambda = \frac{1}{3}\left[1 - \left(1 - \frac{1}{\chi}\right) \cdot\right.$$

$$\left. \cdot\, 2.303\, \frac{\log\left(\Lambda + \frac{1}{\chi}\right)}{\Lambda}\right].$$

A table is compiled for the values of b_{cp} (i.e., $b_{average}$) at two limits Λ and χ; at $\chi = 1$ $b = \frac{1}{3}$ (Table 25).

Table 25

χ \ Λ	0.6	1.0	2.0	3.0	5.0	7.0	10.0
1.1	0.309	0.312	0.316	0.319	0.322	0.324	0.326
1.5	0.246	0.256	0.272	0.282	0.293	0.300	0.306
2.0	0.203	0.218	0.242	0.256	0.273	0.284	0.293
3.0	0.159	0.179	0.211	0.230	0.253	0.267	0.280
4.0	0.137	0.160	0.180	0.280	0.244	0.259	0.273

According to this table the coefficient b increases when Λ increases and χ decreases and tends towards $\frac{1}{3}$ as the limit.

5. WORK DONE IN DISPLACING THE RECOILING PARTS

If we designate the weight and mass of the recoiling parts by Q_0 and M, respectively, and the velocity by V, the work done in moving the recoiling parts will be expressed in the form:

$$E_5 = \frac{MV^2}{2} = \frac{Q_0 V^2}{2g}.$$

The mass M is known, whereas the velocity of recoil V can be found from the theorem of conservation of momentum of the system barrel-charge-projectile which is subjected to the action of internal forces.

However, it is first necessary to find an expression for the absolute velocity of the projectile v_a (with respect to the ground) and for the average velocity of the charge v_ω portions of which move behind the projectile and behind the barrel itself.

Fig. 108 - Diagram of the Distribution of Gas Velocities when the Barrel is Recoiled

1) chamber base; 2) projectile base.

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In order to determine the average absolute velocity v'_ω of the charge, we shall assume, as before, that the mass of the charge is distributed uniformly in the initial air space at each given instant and that the velocity v'_ω changes linearly from V at the base of the chamber to v_a at the base of the projectile, whereby

$$v_a = v - V,$$

where v is the relative velocity of the projectile in the bore of the gun barrel (fig. 108).

If the velocity of the elements of the charge varies linearly, the average velocity will be expressed as the half sum of the end velocities, i.e.,

$$v_{\omega\ cp} = \frac{-V + v_a}{2} = \frac{-V + v - V}{2} = \frac{v}{2} - V.$$

In constructing the equation of momentum and by assuming that the velocities in the direction of the projectile's motion are positive and those in the opposite direction are negative, we will get:

$$-MV + \mu v'_{\omega\ cp} + mv_a = 0,$$

or, replacing $v'_{\omega\ cp}$ and v_a by their values in terms of the relative velocity v of the projectile, we have:

$$-MV + \mu\left(\frac{v}{2} - V\right) + m\,(v - V) = 0,$$

whence

$$V = \frac{m + \frac{\mu}{2}}{M + m + \mu}\, v.$$

Substituting the value of V in the expression for E_5, we get:

$$E_5 = \frac{MV^2}{2} = \frac{M}{2}\,\frac{\left(m + \frac{1}{2}\mu\right)^2}{(M + m + \mu)^2}\, v^2 = \frac{Mm^2}{2M^2}\,\frac{\left(1 + \frac{1}{2}\,\frac{\mu}{m}\right)^2}{\left(1 + \frac{m}{M} + \frac{\mu}{M}\right)^2}\, v^2 =$$

$$= \frac{m}{M}\,\frac{\left(1 + \frac{1}{2}\,\frac{\mu}{m}\right)^2}{\left(1 + \frac{m}{M} + \frac{\mu}{M}\right)^2}\,\frac{mv^2}{2}.$$

Replacing the masses by their weights, we get

$$E_5 = \frac{q}{Q_0} \frac{\left(1 + \frac{1}{2}\frac{\omega}{q}\right)^2}{\left(1 + \frac{q}{Q_0} + \frac{\omega}{Q}\right)^2} \cdot \frac{mv^2}{2}.$$

The factor at $\frac{mv^2}{2}$ is the coefficient k_5:

$$k_5 = \frac{q}{Q_0} \frac{\left(1 + \frac{1}{2}\frac{\omega}{q}\right)^2}{\left(1 + \frac{q}{Q_0} + \frac{\omega}{Q_0}\right)^2}.$$

The factor k_5 depends in the main on the ratio between the weight of the projectile q and the weight of the recoiling parts Q_0; the second factor incorporates only a small change in this ratio. An approximate expression can be used in actual practice. Inasmuch as $\frac{q}{Q_0} + \frac{\omega}{Q_0}$ as well as $\frac{1}{4}\left(\frac{\omega}{q}\right)^2$ are small and approximately equal to each other,

$$k_5 \approx \frac{q}{Q_0}\left(1 + \frac{\omega}{q}\right).$$

If we express $v_{\omega\ cp}$ in terms of the absolute velocity of the projectile v_a, we will get:

$$-MV + \mu\left(\frac{v_a - V}{2}\right) + mv_a = 0,$$

whence

$$V = \frac{m + \frac{1}{2}\mu}{M + \frac{1}{2}\mu} v_a = \frac{q\left(1 + \frac{1}{2}\frac{\omega}{q}\right)}{Q_0\left(1 + \frac{1}{2}\frac{\omega}{Q_0}\right)} v_a .$$

The coefficient k_5 characterizing the relative work of recoil is higher for howitzers than for cannon, because for the same caliber and projectile weight the weight of the recoiling parts Q_0 is considerably smaller on howitzers than on cannon of the same caliber.

6. SUMMATION OF THE AUXILIARY WORK DONE

Upon investigating the auxiliary work items involved and determining a general expression for each in the form

$$E_i = k_i \frac{mv^2}{2} = k_i E_1,$$

an expression can be compiled for expressing the total work done in the energy equilibrium equation:

$$\frac{f\omega}{\theta}\psi - \frac{ps(l_\psi + \ell)}{\theta} = E_1 + E_2 + E_3 + E_4 + E_5 = \Sigma E_i,$$

whereby

$$\Sigma E_i = E_1(1 + k_2 + k_3 + k_4 + k_5).$$

The sum in parenthesis is usually denoted by φ:

$$\varphi = 1 + k_2 + k_3 + k_4 + k_5,$$

or

$$\varphi = 1 + \left(\frac{\rho}{r}\right)^2 \tan^2\alpha + \left(\frac{\rho}{r}\right)^2 \nu_1 \tan\alpha + \frac{1}{3}\frac{\omega}{q} + \frac{q}{Q_0}\left(1 + \frac{\omega}{q}\right).$$

Thus the total exterior work done by the gases when a shot is fired is

$$\Sigma E_i = \frac{\varphi m v^2}{2},$$

where φ is a coefficient representing the auxiliary or secondary work done.

Upon determining φ, when performing the necessary transformations for the solution of the fundamental problem of pyrodynamics, the sum of the work $E_1 \ldots E_5$ in the right side of the equation may be omitted and replaced by the coefficient $\frac{\varphi m v^2}{2}$. φ is the coefficient representing the secondary or auxiliary work items involved. It increases in the main with the increase of the relative weight of the charge ω/q.

In such cases where design data on the gun rifling are not available, φ is calculated by means of simplified formulas.

For example, Prof. V.E. Slukhotsky offers the following general expression for φ:

$$\varphi = K + \frac{1}{3} \frac{\omega}{q},$$

by introducing into K the sum of all the k_i's except k_4, which is separately expressed by $\frac{1}{3} \frac{\omega}{q}$. The value of K varies with the type of gun:

for howitzers........................K = 1.06
for medium-power cannon..............K = 1.04-1.05
for high-power cannon................K = 1.03
for small arms.......................K = 1.10

The difference in the value of the K coefficient for cannon and howitzers is mainly obtained because of the work done by the recoil (k_5 coefficient):

Sugot, offers the following expression for φ:

$$\varphi = 1.05 \cdot \left(1 + \frac{1}{4}\frac{\omega}{q}\right).$$

CHAPTER 6 - SUPPLEMENTARY PROBLEMS

1. RELATION BETWEEN THE PRESSURES EXERTED ON THE BASE OF THE BORE AND THE BASE OF THE PROJECTILE

When the projectile moves under the action of the powder gases, the pressure in the initial air space is not uniform because the gases are in motion and move with varying rates of speed.

In order to determine the pressure exerted on the base of the shell and the base of the bore, let us consider two sections of the initial air space: at base of the bore and the base of the shell, and construct an equation of motion of the masses located to one side (to the right) of these sections.

Then the projectile of mass m under the action of force $p_{CH}s$ will be subjected to an acceleration j, and the equation of motion for the section at the base of the projectile, with the resisting forces taken into account, will be written as follows:

$$p_{CH}s = \varphi_1 mj.$$

The gas pressure $p_{ДH}$ at the base of the shell will impart an accelerated motion not only to the shell, but also to the entire gas mass to the right of this section, between the base of the bore and the base of the projectile. Therefore the equation of motion at this section will be written thus:

$$p_{ДH}s = \varphi_1 mj + \mu j_3,$$

where μ - mass of gases generated by the charge (including the still unburned portion of the powder);

j_3 - mean acceleration of the charge moving with a varying velocity from one layer to another.

Dividing these equations term by term, we get:

$$p_{\text{дн}} = p_{\text{сн}}\left(1 + \frac{\mu j_3}{\varphi_1 m j}\right) = p_{\text{сн}}\left(1 + \frac{j_3 \omega}{j \varphi_1 q}\right).$$

The ratio between the accelerations developed by the charge and the projectile is governed by the hypothesis applied with regard to the distribution of the mass of the charge in the initial air space and by the law of the change of accelerations at the various sections of the space.

If the law governing the gas velocity change is linear, the ratio $j_3/j = \xi$ equals 0.5 and this magnitude, usually called the Piober coefficient, is the one used in most textbooks on the subject.

The problem dealing with the distribution of gas pressure in the initial air space is analyzed in great detail by Prof. I.P. Gravé [18] and, recently, by Asst. Prof. P.N. Shkvornikov [7] who gives a solution to this problem based on the assumptions commonly used in gas dynamics with relation to gas motion under condition of uniform density of the mixture of gas and unburned portion of powder at a given position of the projectile in the bore. All the relations are derived from the fundamental equations of motion of the powder gases and the burning charge which, in turn, are obtained from the general equations of gas dynamics for a one-dimensional unstable gas motion.

The fundamental formulas developed by P.N. Shkvornikov follow. Using the designations:

v - relative velocity of projectile;

v_a - absolute velocity of projectile;

V - velocity of recoiling parts;

q - weight of projectile;

Q_0 - weight of recoiling parts;

φ_1 and φ_2 - coefficients representing the resistances encountered by the projectile in moving through the gun bore and by the recoiling parts.

Introducing the designation

$$i = \frac{\varphi_1 q + \frac{1}{2}\omega}{\varphi_2 Q_0 + \frac{1}{2}\omega};$$

then

$$V = iv_a = \frac{i}{1 + i}v$$

and

$$v_a = \frac{1}{1 + i}v.$$

The relation between p_{AH} and p_{CH}(*) will be expressed by the formula:

(*) p_{AH} - pressure at base of bore; p_{CH} - pressure at base of projectile.

$$p_{дн} = p_{сн}\left[1 + 0.5\,\frac{\omega}{\varphi_1 q}\,(1 - i)\right].$$

The maximum pressure develops not at the base of the bore but, rather, at a distance $x_m = \frac{i}{1 + i}\,\Lambda$ at the section where the absolute velocity of the powder gases is $v_a = 0$.

In deriving the fundamental equation of pyrodynamics and assuming that $pw = RT\omega\psi$, the pressure p was understood to represent a certain mean pressure which is identical at all points in the initial air space; it is assumed thereby that the powder burns under precisely such a pressure. The relation between p, $p_{сн}$ and $p_{дн}$ is given in the following form:

$$p = p_{сн}\left[1 + \frac{1}{3}\,\frac{\omega}{\varphi_1 q}\,\left(1 - \frac{i}{2}\right)\right];$$

$$p = p_{дн}\,\frac{1 + \frac{1}{3}\,\frac{\omega}{\varphi_1 q}\left(1 - \frac{i}{2}\right)}{1 + \frac{1}{2}\,\frac{\omega}{\varphi_1 q}(1 - i)}.$$

For a recoilless barrel $i = 0$ and the formulas are simplified:

$$p = p_{сн}\left(1 + \frac{1}{3}\,\frac{\omega}{\varphi_1 q}\right).$$

Multiplying both parts of this equation by φ_1, we get:

$$\varphi_1 p = p_{сн}\left(\varphi_1 + \frac{1}{3}\,\frac{\omega}{q}\right) = p_{сн}(1 + k_2 + k_3 + k_4).$$

Hence, if we disregard the work done in recoiling the barrel,

$$\varphi_1 p \approx \varphi p_{CH} \quad \text{or} \quad \frac{p_{CH}}{\varphi_1} = \frac{p}{\varphi}.$$

In such a case the equation of the projectile's motion

$$sp_{CH} = \varphi_1 m \frac{dv}{dt}$$

will be equal to the equation of the projectile's motion

$$sp = \varphi m \frac{dv}{dt},$$

where p - mean gas pressure;

φ - a coefficient which takes into account all the auxiliary or secondary work done.

This constitutes a very important deduction, because it permits to consider p and φ to be identical values in both the fundamental equations of pyrodynamics

$$ps(\ell_\psi + \ell) = f\omega\psi - \frac{\theta\varphi m v^2}{2}$$

and in the equation of the projectile's motion

$$ps = \varphi m \frac{dv}{dt} \quad \text{or} \quad ps = \varphi m v \frac{dv}{d\ell}$$

whereby v is the relative velocity of the projectile (relative to the bore), which is computed in solving the problem of interior ballistics.

For cannon and howitzers $\varphi_1 = 1.02$, for small arms using ordinary bullets $\varphi_1 = 1.05$, for armor-piercing bullets $\varphi_1 = 1.07$.

The coefficient i depending on the distribution of the moving mass in the system barrel-charge-projectile, may be considered to be equal to:

$i = 0.0015$ for rifles and machine guns.

$i = 0.0030$ for antitank guns

$i = 0.020$ for cannon

$i = 0.035$ for howitzers

This data was arrived at from the analysis of the results obtained by P.N. Shkvornikov for some of our (Russian) artillery systems and small arms.

2. HEAT LOST TO THE BARREL WALLS THROUGH HEAT TRANSFER

When considering the heat lost to the walls of the barrel when a gun is fired, we shall take into account the heat transfer resulting from the direct contact of the hot gases with the cold walls of the gun barrel; we shall not consider the heating of the walls due to mechanical reasons (energy of translation of projectile, friction by the rotating band, and deformation of barrel).

The data entering these calculations were presented in Part I of this book. The basic allowance used is the same as that assumed in computing the heat transferred to the walls of a manometric bomb, namely, that the heat loss is proportional to the number of impacts made by the molecules against the wall of the bomb, which in turn depends on Σ, p and t, i.e., on $\Sigma\int pdt$, where Σ is the surface area of the bore.

However, whereas the area $S_б$ remains constant in a bomb, Σ varies from the chamber area Σ_0 to area Σ_{KH} of the entire barrel surface. The chamber area is constantly subjected to the action of the gases and thus takes up a portion of the heat energy, whereas the area of the rifled portion of the bore participates in the cooling of the gases only while the projectile is in motion and thus gradually tends to increase the initial air space. The nearer the rifled portion to the muzzle, the shorter will be the period during which it will be subjected to the action of the gases, and the less heat will it take up when a shot is fired.

After the projectile's departure, the heat transfer to the walls continues along the entire area of the bore at a constantly decreasing pressure, but inasmuch as this heat transfer no longer affects the actual shot (reduced pressure and velocity), we shall not consider it for the time being.

In order to take into account the effect of heat transfer when a shot is fired, it is first necessary to determine by means of bomb tests the time t_K it takes for the powder to burn at $\Delta = 0.20$, and also the coefficient C_K% from the Muiraur C curve.

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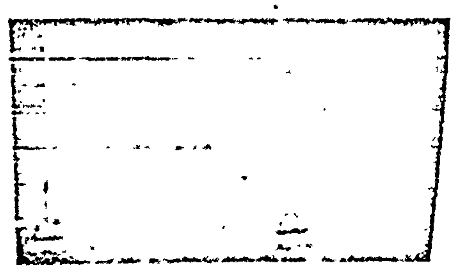

Fig. 109 - Heat-Transfer Diagram According to Miuraur

If the powder were burned in the chamber at constant initial charge density Δ_0, the correction for heat transfer would be determined by the following formula:

$$\left(\frac{\Delta T_{\%}}{T}\right)_0 = \frac{C_M}{7.774}\frac{\Sigma_0}{W_0}\frac{1}{\Delta_0},$$

where Σ_0 is the chamber surface area and $\frac{\Sigma_0}{W_0} = \frac{4}{D} + \frac{1}{\ell_{KM}}$;

ℓ_{KM} is the length, and D is the mean diameter of the chamber.

If the powder had burned all the time throughout the entire bore space at a loading density of $\Delta_{KH} = \frac{\omega}{W_0 + s\ell_Д} = \frac{\omega}{W_{KH}}$, the loss would be expressed by the formula:

$$\left(\frac{\Delta T_{\%}}{T}\right)_Д = \frac{C_M}{7.774}\frac{\Sigma_{KH}}{W_{KH}}\frac{1}{\Delta_{KH}}, \qquad (70)$$

where Σ_{KH} is the surface area of the whole bore.

Actually, the loss suffered when a shot is fired is confined between these two values, and in order to account for same when the pressure and the cooling surface of the bore change simultaneously, these changes whose increments are proportional to the path traversed by the projectile must be known as a function of time.

With the p, t and Σ, t or ℓ, t curves at his disposal, Muiraur suggested the following method for determining losses due to heat transfer.

The p, t (I) and $\frac{\Sigma}{\Sigma_{KH}}$, t(II) curves are plotted on the same diagram (fig. 109); point C' represents the end of burning of the powder ($p = p_K$).

The area of the pressure curve $Op_0p_mp_KC'O$ corresponds to $I_K = \int_0^{t_K} pdt$ obtained from bomb tests; the area $C'p_Kp_ДC'' = I_{II}$ corresponds to the additional cooling which would obtain in the second period after the end of burning of the powder just before the projectile leaves the barrel.

If the surface area were constant, the right side of the equation would have to be multiplied by the area ratio in order to take the heat transfer into account:

$$\frac{Op_0p_mp_Д C''O}{Op_0p_mp_K C'O} = \frac{I_K + I_{II}}{I_K} > 1.$$

But since the area varies from the relative value $\frac{\Sigma_0}{\Sigma_{KH}}$ to $1 = \frac{\Sigma_{KH}}{\Sigma_{KH}}$ according to the law expressed by curve II, a correction must be introduced into formula (70) by multiplying the ordinates of curve I by the ordinates of curve II; the obtained products are given in the form of curve III.

The ratio of the area $OABp_Д C''O = \int \frac{\Sigma}{\Sigma_{KH}} pdt$ to I_K will then show the part of the losses computed by means of formula (70) that must be taken in order to determine the loss obtained due to the simultaneous change of the area and the pressure. Miuraur had found that this ratio must be equal to 0.43÷0.46.

Thus the loss due to heat transfer and the relative drop in temperature due to this loss will be expressed by the following formula:

$$\frac{\Delta T}{T}\% = \frac{C_M}{7.774} \frac{\Sigma_{KH}}{W_{KH}} \frac{1}{\Delta_Д} \frac{1}{I_K} \int_0^t \frac{\Sigma}{\Sigma_{KH}} pdt. \qquad (71)$$

The losses computed in this manner for various weapons amount to

1% in the case of a 152-mm cannon and to about 15% for a rifle.

Therefore cooling through the walls when a shot is fired varies within very wide limits and amounts to from 1 to 15% of the total heat energy; thus whereas an error of 1% could be disregarded, no such allowance can be made in the case of a 15% error, and the latter error must therefore be accounted for.

The method used by Miuraur for determining losses incurred during a shot requires the availability of pressure and distance or path curves as a function of time, which data are usually obtained by means of numerical integration or from АНИИ (ANII) or ГАУ(GAU) tables.

The author had shown that the heat lost to the walls of the barrel can be calculated also in the absence of the p and ℓ curves as a function of time.

Bearing in mind that $W_{KH}\Delta_{KH} = W_0\Delta_0 = \omega$, pdt = dI, the above formula can be rewritten thus:

$$\left(\frac{\Delta T}{T}\right)\% = \frac{c_M}{7.774}\,\frac{\Sigma_{KH}}{\omega}\int_0^I \frac{\Sigma dI}{\Sigma_{KH} I_K} = \frac{c_M}{7.774}\,\frac{\Sigma_0}{\omega}\int_0^I \frac{\Sigma}{\Sigma_0}\,\frac{dI}{I_K}. \qquad (72)$$

But from the equation of motion

$$\varphi m dv = spdt = sdI,$$

whence

$$dI = \frac{\varphi m}{s}dv, \quad I_K = \frac{\varphi m}{s}v_K' = \frac{\varphi m}{s}(v_0' + v_K),$$

where v_0' is the velocity the projectile would have attained if it had started to move as if there were no rifling at the instant a portion of the charge ψ_0 is burned, corresponding to the true pressure:

$$v'_0 = \frac{s}{\varphi m} I_0 = \frac{s}{\varphi m} \int_0^{\psi_0} p dt.$$

This value serves as a means for determining the heat lost in the chamber due to transfer before the projectile starts moving. The ratio $\frac{dI}{I_K}$ can be replaced by the ratio

$$\frac{dv}{v'_K} = \frac{dv}{v'_0 + v_K}.$$

$\sum_{KH}$ represents the total surface area of the chamber $\sum_0$ and of the rifled portion of the bore $\pi d'' l_{\text{Д}}$, where d" is the reduced diameter of the circle having the same perimeter as the perimeter of the cross section of the bore including the rifling bores. If the number of grooves is n, and their depth is t_H,

$$\pi d'' = \pi (d + t_H) + 2nt_H = \pi d \left[1 + \frac{t_H}{d} \left(1 + \frac{2n}{\pi} \right) \right].$$

Inasmuch as in the barrels of artillery pieces, $t_H = 0.01\text{-}0.02d$,

$$d'' = d[1 + (0.01...0.02)(1 + 0.64n)]$$

when n = 28.

$$d'' = d[1 + (0.01...0.02)18.9] = d[1.19...1.38] = d[1 + \alpha_1].$$

The numerals in parenthesis show that the increase of the cooling surface due to the presence of rifling is very great and becomes greater with the increase of n and t_n/d.

Inasmuch as the mean diameter of the powder chamber is

$$D = d\sqrt{\chi},$$

where $\chi = \dfrac{\ell_0}{\ell_{KM}}$ is the widening coefficient of the chamber greater than unity,

$$\Sigma_0 = \pi d \sqrt{\chi}\,\ell_{KM} + 2\,\frac{\pi}{4}d^{"2} = \pi d \ell_{KM}\left[\sqrt{\chi} + \frac{1}{2}\,\frac{d}{\ell_{KM}}\,[1 + \alpha_1]\right].$$

Assuming for the sake of simplicity that

$$\ell_{KM}\left[\sqrt{\chi} + \frac{1}{2}\,\frac{d^{"}}{\ell_{KM}}\,[1 + \alpha_1]\right] \approx \ell_0\,[1 + \alpha_1],$$

where $\ell_0 = \dfrac{W_0}{s}$ is the reduced length of the chamber, we obtain expressions for the areas of the bore at the start, at the end and at an intermediate instant:

$$\Sigma_0 = \pi d(1 + \alpha_1)\,\ell_0;$$

$$\Sigma_{KH} = \pi d(1 + \alpha_1)(\ell_0 + \ell_{Д});$$

$$\Sigma = \pi d(1 + \alpha_1)(\ell_0 + \ell);$$

$$\frac{\Sigma}{\Sigma_0} = \frac{\ell_0 + \ell}{\ell_0} = 1 + \frac{\ell}{\ell_0};\quad \frac{\Sigma}{\Sigma_{KH}} = \frac{\ell_0 + \ell}{\ell_0 + \ell_{Д}}.$$

Replacing $\dfrac{dI}{I_K}$ and $\dfrac{\Sigma}{\Sigma_0}$ in formula (72) by their corresponding expressions, we get a relationship for the heat transfer occurring while the projectile moves through the bore in the form:

$$\frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\int_0^1 \frac{\Sigma}{\Sigma_0}\,\frac{dI}{I_{\text{К}}} = \frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\int_0 \frac{(\ell_0 + \ell)\,dv}{\ell_0(v_0' + v_{\text{К}})}. \tag{73}$$

To this must be added the heat losses in the chamber during the preliminary period, determined by the analogous formula

$$\frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{I_0}{I_{\text{К}}} = \frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{\ell_0 v_0'}{\ell_0(v_0' + v_{\text{К}})}. \tag{74}$$

Adding formulas (73) and (74), we get an expression for the gas temperature drop occurring while the projectile is in motion due to heat lost to the walls:

$$\left(\frac{\Delta T}{T}\right)\% = \frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{\ell_0(v_0' + v) + \int_0^{v_{\text{Д}}} \ell\,dv}{{}_0(v_0' + v_{\text{К}})} = \frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\,\eta. \tag{75}$$

Prior to the start of motion, $v = 0$, and expression (75) is transformed into (70). Prior to the end of travel in the bore, $v = v_{\text{Д}}$, so that:

$$\left(\frac{\Delta T}{T}\right)_{\text{Д}}\% = \frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\,\frac{\ell_0(v_0' + v_{\text{Д}}) + \int_0^{v_{\text{Д}}} \ell\,dv}{\ell_0(v_0' + v_{\text{К}})} = \frac{C_{\text{М}}}{7.774}\,\frac{\Sigma_0}{\omega}\,\eta_{\text{Д}}. \tag{76}$$

A curve depicting the velocities of the projectile as a function of the traversed path ℓ is adequate for the purpose of calculating this loss.

The ℓ, v diagram in fig. 110 constructed by the author indicates that $\eta_{\text{Д}}$ represents a ratio of the areas

$$\eta_{\text{Д}} = \frac{\text{area aoefhda}}{\text{area abcd}};$$

the rectangle aghd characterizes the loss incurred in the chamber, the area of the curvilinear figure oefg represents the loss in the rifled portion of the barrel, and the rectangle aokd represents the loss in the preliminary period.

The factor $\eta_{\text{Д}}$ represents the ratio between the total heat lost to the walls of the entire bore, which takes into account the fact that the rifled portion enters into action gradually, as the projectile moves through it, and the heat lost in the chamber at the end of burning of the powder; the area of the chamber includes the area of the base of the projectile.

By analogy, the coefficient η characterizes the heat transfer at a given time.

When computing the cross-hatched area in fig. 110, v is the independent variable and ℓ is the dependent one. But the situation will not change if we were to turn the graph around in such a way as to obtain an ordinary curve of velocities v as a function of the path ℓ. In such a case we would have a v, ℓ graph (fig. 111) instead of an ℓ, v graph, wherein the cross-hatched area lies above the v, ℓ curve. The numerical values of η and $\eta_{\text{Д}}$ will remain the same, but due to the change of the coordinates the expression for the area in the numerator will be different:

$$\eta = \frac{\ell_0(v_0' + v) + v\ell - \int_0^{\ell} v\,d\ell}{\ell_0(v_0' + v_K)};$$

$$\eta_{Д} = \frac{\ell_0(v_0' + v_{Д}) + v_{Д}\ell_{Д} - \int_0^{\ell_{Д}} v\,d\ell}{\ell_0(v_0' + v_K)}.$$

The relation between v and ℓ can be found in the ANII or GAU tables by means of the ratio

$$\frac{\ell}{\ell_0} = \Lambda.$$

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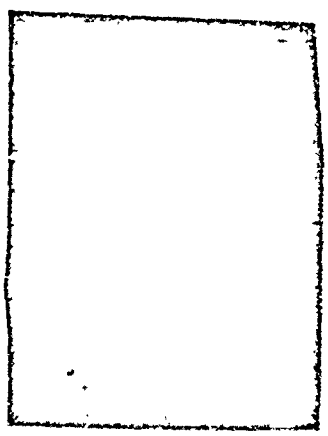

Fig. 110 - Diagram for Computing the Heat Transfer in Terms of ℓ, v Coordinates

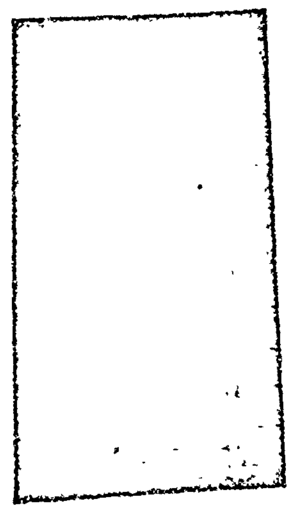

Fig. 111 - Diagram for Computing the Heat Transfer in Terms of v, ℓ Coordinates

Dividing the numerator and denominator of η by ℓ_0, disregarding the value of v_0' in the numerator, and bearing in mind that

$$v_0' + v_K = \frac{s l_K}{\varphi_{ш}} = v_1,$$

we get

$$\eta = \frac{v(1+\Lambda) - \int_0^{\lambda} v d\lambda}{v_1} = (1+\Lambda)\frac{v}{v_1} - \int_0^{\lambda} \frac{v}{v_1} d\Lambda;$$

$$\frac{v}{v_1} = \frac{v_{table}\sqrt{\varphi\frac{\omega}{q}\varphi_m}}{sI_K} = v_{table}\sqrt{\frac{\omega\varphi^2 m^2}{mgs^2 I_K^2}} = v_{table}\sqrt{\frac{f\omega\varphi m}{s^2 I_K^2 fg}} =$$

$$= \sqrt{\frac{1}{B}}\,\frac{v_{table\,\text{д}}}{\sqrt{fg}}.$$

On the basis of the constants assumed in the ANII tables 95,000 kg-m/kg; $g = 9.81$ m/sec^2; $\varphi = 1.05$; $\sqrt{fg} = 955$ and $\frac{v}{v_1} =$ $= \frac{v_{table}\ \text{m/sec}}{955\sqrt{B}}$.

Therefore

$$\eta = \frac{1}{955\sqrt{B}}\left[(1+\Lambda)v_{table} - \int_0^{\Lambda} v_{table}\, d\Lambda\right];$$

$$\eta_{\text{д}} = \frac{1}{955\sqrt{B}}\left[(1+\Lambda)v_{table} - \int_0^{\Lambda_{\text{д}}} v_{table}\, d\Lambda\right].$$

Incorporating these expressions in formulas (75) and (76), we will find the heat transfer losses incurred when a shot is fired from the ANII tables.

It should be noted that when calculating $\frac{\Sigma_0}{\omega} - \frac{\Sigma_0}{W_0 \Delta_0}$, the Σ_0 and W_0 must be expressed in cm^2 and cm^3.

Results of Calculations. Calculations for a 76-mm cannon of 1902 issue measuring 30 calibers in length show that for a normal charge $\left(\frac{\Delta T}{T}\right)$ % ≈ 1.8; when the charge is reduced the loss increases somewhat and amounts to 2.2%. Upon increasing the length of the barrel to 50 calibers the loss incurred with a normal charge equals 2.5%, and is 3.1% for a reduced charge.

If expression (76) is presented in the form

$$\left(\frac{\Delta T}{T}\right)_Д \% = K_T \eta_Д,$$

where

$$K_T = \frac{C_M}{7.774} \frac{\Sigma_0}{W_0} \frac{1}{\Delta},$$

we shall have the following data (Table 26) for a 76-mm cannon of 1900 issue, using СП strip powder ($2e_1$ = 1.03 mm) (based on N.A. Zabudsky's tests) and C_M = 4.6%, when varying the charge.

Table 26

ω	1.041	0.892	0.725	0.558
Δ	0.613	0.525	0.426	0.328
K_T	0.553	0.646	0.795	1.034
η_K	1.36	1.51	1.75	2.24
$\eta_Д$	3.01	2.83	2.54	2.11
$\left(\frac{\Delta T}{T}\right)_K \%$	0.75	0.98	1.39	2.32
$\left(\frac{\Delta T}{T}\right)_Д \%$	1.67	1.83	2.00	2.18
$\ell_K + \ell_Д$	7.79	9.98	14.23	25.05

This table shows that when the charge is decreased, the heat transfer loss increases from 1.7 to 2.2%. When the barrel length is increased to 50 calibers, the percentage value increases correspondingly to 2.3 and 3.1%. When the charge is decreased the drop in temperature at the end of burning becomes greater due to the increased cooling surface area, because the end of burning is transposed towards the muzzle (see line $\ell_K + \ell_0$).

Calculations for systems of different calibers have shown that when the caliber is increased the heat transfer losses decrease because of the reduced value of $\frac{\Sigma_0}{W_0}$, and notwithstanding the increase of C_M which increases slowly in the case of thick powders, η_n fluctuates between the limits of 2.25 and 3.75. The heat loss in a rifle amounts to 10% and must be accounted for by reducing correspondingly the powder energy $f = RT_1$, i.e., as if the burning temperature of the powder were reduced. For guns varying from 37 to 76 mm in caliber, the energy f determined in bomb without corrections for heat transfer may be considered acceptable, because the heat losses obtained in such guns and in bombs are about the same (2-3%).